4.5 HW

Qs: ~~1~~, ~~3~~, ~~7~~, ~~11~~, ~~15~~, ~~22~~, ~~29~~, ~~33~~, ~~37~~, ~~39~~, ~~41~~, ~~45~~, ~~47~~, ~~51~~, ~~59~~, ~~61~~, ~~65~~, ~~71~~

① $\int (8x^2+1)^2(16x)\,dx$ $\quad \boxed{u=8x^2+1} \quad \boxed{du=16x\,dx}$

③ $\int \tan^2 x \sec^2 x\,dx$ $\quad \boxed{u=\tan x} \quad \boxed{du=\sec^2 x\,dx}$

⑦ $\int \sqrt{25-x^2}\,(-2x)\,dx$ $\quad u=25-x^2 \quad du=-2x\,dx$

$$\int u^{1/2}\,du \to \frac{u^{3/2}}{3/2}+C \to \frac{(25-x^2)^{3/2}}{3/2}+C$$

$$\to \boxed{\frac{2(25-x^2)^{3/2}}{3}+C}$$

⑪ $\int x^2(x^3-1)^4\,dx \to$ $\quad u=x^3-1 \quad du=3x^2\,dx$

$\hookrightarrow 3x^2$ & $dx \nearrow$

$$\frac{1}{3}\int u\,du$$

$$\to \frac{1}{3}\left(\frac{u^5}{5}\right)+C \to \boxed{\frac{(x^3-1)^5}{15}+C}$$

⑮ $\int 5x\sqrt[3]{1-x^2}\,dx \to u=1-x^2 \to du=-2x\,dx$

$\hookrightarrow x\,dx=-\frac{1}{2}du$

$$\to 5x\,dx \to 5(x\,dx) \to 5\left(-\frac{1}{2}du\right) \to = -\frac{5}{2}du$$

$$\to -\frac{5}{2}\int u^{1/3}\,du \to -\frac{5}{2}\left(\frac{3u^{4/3}}{4}\right)+C$$

$$\to \boxed{-\frac{15}{8}(1-x^2)^{4/3}+C}$$

㉒ $\int \frac{x^3}{\sqrt{1+x^4}}\,dx \to$ $\quad u=1+x^4 \quad du=4x^3\,dx$

$\to (1+x^4)^{-1/2}$ $\quad \hookrightarrow 4x^3$ & dx

$$\to \frac{1}{4}\int u^{-1/2}\,du = \frac{1}{4}\left(\frac{2u^{1/2}}{1}\right)+C \to \boxed{\frac{1}{2}(1+x^4)+C}$$

$$\frac{du}{dx}=4x^3 \to du=4x^3\,dx \to x^3\,dx=\frac{du}{4}$$

㉙ $\frac{dy}{dx} = \frac{x+1}{(x^2+2x-3)^2} \rightarrow u = x^2+2x-3$

$\frac{du}{dx} = 2x+2 \rightarrow du = (2x+2)\,dx \rightarrow x+1\,dx = \frac{1}{2}du$

$\frac{1}{2}\int u^{-2}\,du = \frac{1}{2}\left(\frac{u^{-1}}{-1}\right) + C \rightarrow \frac{1}{2}\left(\frac{1}{(-1)(x^2+2x-3)}\right) + C$

$\rightarrow \frac{1}{2}\left(\frac{1}{-x^2-2x+3}\right) + C \rightarrow \boxed{\frac{-1}{2(x^2+2x-3)} + C}$

㉝ $\int \pi \sin(\pi x)\,dx \rightarrow u = \pi x \quad du = \pi\,dx$

$\leftrightarrow \pi\,dx$

$\rightarrow \int \sin u\,du \rightarrow -\cos u + C \rightarrow \boxed{-\cos(\pi x) + C}$

㊲ $\int \frac{1}{\theta^2}\cos\frac{1}{\theta}\,d\theta \rightarrow \quad u = \frac{1}{\theta} \quad du = -\theta^{-2}\,d\theta$

$\leftrightarrow \theta^{-1} = -1\theta^{-2} \,\&\, d\theta$

$\rightarrow -1\int \cos u\,du \rightarrow -1(\sin u) + C \rightarrow \boxed{-\sin\frac{1}{\theta} + C}$

㊴ $\int \sin 2x \cos 2x\,dx \qquad u = \sin 2x \qquad du = 2\cos 2x\,dx$

$\leftrightarrow 2\cos 2x$

$\rightarrow \int u\,\frac{du}{2} \rightarrow \frac{1}{2}\int u\,du \rightarrow \frac{1}{2}\left[\frac{u^2}{2}\right] + C \rightarrow \frac{u^2}{4} + C$

$\rightarrow \boxed{\frac{(\sin 2x)^2}{4} + C}$

㊶ $\int \frac{\csc^2 x}{\cot^3 x}\,dx \rightarrow \frac{\left(\frac{1}{\sin^2 x}\right)}{\left(\frac{\cos^3 x}{\sin^3 x}\right)} \rightarrow \frac{1}{\sin^2 x} \cdot \frac{\sin^3 x}{\cos^3 x}$

$\rightarrow \frac{\sin x}{\cos^3 x} \qquad u = \cos x \quad \rightarrow du = -\sin x\,dx$

$\rightarrow \sin x\,dx = -du$

$\rightarrow \int \frac{\sin x}{\cos^3 x}\,dx \rightarrow \int \frac{1}{u^3}(\sin(x)\,dx) = \int \frac{1}{u^3}(-du)$

$\rightarrow -\int u^{-3}\,du \rightarrow -\left[\frac{u^{-2}}{-2}\right] + C \rightarrow \frac{u^{-2}}{2} + C$

$\rightarrow \boxed{\frac{1}{2\cos^2 x} + C}$

(45) $f'(x) = 2x(4x^2-10)^2$ point: $(2, 10)$

$u = 4x^2 - 10$ $\frac{du}{dx} = 8x \to du = 8x\,dx \to x\,dx = \frac{du}{8}$

$2x\,dx = 2(x\,dx) = 2\left(\frac{du}{8}\right) \to \frac{1}{4}\,du$

$\to \int 2x(4x^2-10)^2\,dx \to \int (u^2)\frac{1}{4}\,du \to \frac{1}{4}\int u^2\,du$

$\to \frac{1}{4}\left[\frac{u^3}{3}\right]+C \to \frac{u^3}{12}+C \to \frac{(4x^2-10)^3}{12}+C$

$\to F(x) = \frac{(4x^2-10)^3}{12}+C$ $y(2)=10$

$10 = \frac{(4(2)^2-10)^3}{12}+C \to \frac{(16-10)^3}{12}+C \to \frac{6^3}{12}+C=10$

$\to \frac{216}{12}+C=10 \to 18+C=10 \to C=-8$

$\to \boxed{y(x) = \frac{(4x^2+10)^3}{12}-8}$

(47) $\int x\sqrt{x+6}\,dx$ $u=x+6$ $\frac{du}{dx}=1 \to du=dx$

$x=u-6$ $\int x\sqrt{x+6}\,dx \to \int (u-6)u^{1/2}\,du$

$\to \int \left(u^{3/2}-6u^{1/2}\right)du \to \left(\frac{2u^{5/2}}{5} - \frac{6u^{3/2}}{1}\cdot\frac{2}{3}\right)+C$

$\to \left(\frac{2u^{5/2}}{5}-\frac{12u^{3/2}}{3}\right)+C \to \left(\frac{2(x+6)^{5/2}}{5}-\frac{12(x+6)^{3/2}}{3}\right)$

$+C \to \frac{2}{5}(x+6)^{5/2} - 4(x+6)^{3/2}+C$

(51) $\int \frac{x^2-1}{\sqrt{2x-1}}\,dx$ $u=\sqrt{2x-1}$ $u^2=2x-1$

$\to u^2+1=2x \to x=\frac{u^2+1}{2}$ $\frac{d}{dx}(u^2)=(2x-1)\frac{d}{dx}$

$\to 2u\frac{du}{dx}=2 \to \frac{du}{dx}=\frac{1}{u} \to du=\frac{dx}{u} \to dx=u\,du$

$\to x^2-1 \to x=\frac{u^2-1}{2} \to x^2=\frac{(u^2-1)^2}{4} \to x^2=\frac{u^4+2u^2+1}{4}$

$x^2-1 = \frac{(u^4+2u^2+1)}{4} - 1 \to x^2-1 = \frac{u^4+2u^2+1-4}{4}$

$\to x^2-1 = \frac{u^4+2u^2-3}{4}$ $\quad dx = u\,du$

$\to \frac{x^2-1}{\sqrt{2x-1}} = \frac{\frac{(u^4+2u^2-3)}{4}}{u} \to \frac{u^4+2u^2-3}{4u}\,du$

$\frac{1}{4}\int u^4+2u^2-3\ du \to \frac{1}{4}\left[\left(\int u^4\,du\right)+\left(2\int u^2\,du\right)-\left(3\int 1\,du\right)\right]$

$\to \frac{1}{4}\left[\frac{u^5}{5}+\frac{2u^3}{3}-3(u)\right]+C \to \frac{u^5}{20}+\frac{u^3}{6}-\frac{3u}{4}+C$

$(2x-1)^{5/2} \quad (2x-1)^{3/2} \quad (\sqrt{2x-1})$

$\to \boxed{\frac{(2x-1)^{5/2}}{20}+\frac{(2x-1)^{3/2}}{6}-\frac{3(2x-1)^{1/2}}{4}+C}$

(59) $\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx \to u=\sqrt{2x+1} \quad u^2=2x+1$

$u^2=2x+1 \to 2u\frac{du}{dx}=2 \to \frac{du}{dx}=\frac{1}{u} \to dx=u\,du$

$\to \int\frac{1}{\sqrt{2x+1}}\,dx = \int\frac{1}{u}(u\,du) = \int 1\,du = u+C \to \sqrt{2x+1}+C$

$\left[\sqrt{2x+1}\right]_0^4 \to \left(\sqrt{2(4)+1}\right)-\left(\sqrt{2(0)+1}\right) \to \sqrt{9}-\sqrt{1}$

$= \boxed{2}$

(61) $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx \to u=\sqrt{x} \quad \frac{du}{dx}=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$

$\int_1^9 \frac{1}{u}\left(\frac{1}{(1+u)^2}\right)2u\,du \to \int_1^9 \frac{2}{(1+u)^2}\,du$

u-bounds:
$x=1$
$\to 1$
$x=9$
$\to 3$

$\to\left[-\frac{2}{1+u}\right]_1^3 \to \left(-\frac{2}{1+1}\right)-\left(-\frac{2}{1+3}\right) \to -1-\left(-\frac{1}{2}\right)$

$\to -1+\frac{1}{2} \to \boxed{\frac{1}{2}}$

(65) $\int_0^7 x\sqrt[3]{x+1}\,dx \quad u=x+1 \to \frac{du}{dx}=1 \to du=dx$

$\to x=u-1 \to \int_0^7 u-1(u)^{1/3}\,du \to \int_0^7 u^{7/3}-u^{1/3}\,du$

$$\int_0^7 u^{7/3} - u^{1/3}\,du \to \left[\frac{3u^{10/3}}{10} - \frac{3u^{4/3}}{4}\right]_0^7$$

$$\to \left[\frac{3\sqrt{x+1}^{10/3}}{10} - \frac{3\sqrt{x+1}^{4/3}}{4}\right]_0^7 \to$$

u-bounds:
$x=1$
$u\ 1$
$x=7$
$u\ \sqrt{8} \to 2\sqrt{2}$

$\to \boxed{43.179}$

(77) $\int x(5-x^2)^3\,dx \neq \int u^3\,du$

these are not equal because: if $u = 5-x^2$, du must equal $-2x\,dx$, and $\int u^3\,du$ does not provide for the $\boxed{-2}$

4.2 HW

Q. 3, ~~7~~, ~~9~~, ~~17~~, ~~25~~, ~~27~~, 31, ~~35~~, ~~39~~, ~~41~~, ~~47~~, ~~53~~

③ $\sum_{k=0}^{4}\frac{1}{k^2+1} \to \left(\frac{1}{0+1}\right)+\left(\frac{1}{1^2+1}\right)+\left(\frac{1}{2^2+1}\right)+\left(\frac{1}{3^2+1}\right)+\left(\frac{1}{4^2+1}\right) \to 1+\frac{1}{2}+\frac{1}{10}+\frac{1}{17}+\frac{1}{5} \to$

$\to \frac{1}{17}\cdot 2 \to \frac{2}{34}\cdot 5 \to \frac{10}{170}+\frac{170}{170}+\frac{17}{170}+\frac{85}{170}+\frac{34}{170}$

$\to 316/170 \to \boxed{\frac{158}{85}}$

⑦ $\boxed{\sum_{i=1}^{11}\left(\frac{1}{5(i)}\right)}$

⑨ $\boxed{\sum_{i=1}^{6}\left(7\left(\frac{i}{6}\right)+5\right)}$

⑰ $\sum_{i=1}^{20}(i-1)^2 \to \sum_{k=0}^{19}k^2 \to \frac{19(19+1)(2(19)+1)}{6} \to \frac{19(20)(39)}{6}$

$\to \boxed{2470}$

㉕ $f(x)=2x+5$, $[0,2]$, 4 partitions, find inscribed & circumscribed

lower $\frac{1}{2}(f(0)+f(0.5)+f(1)+f(1.5))$

$\to \frac{1}{2}(5+6+7+8) \to \frac{1}{2}(26) \to \boxed{13}$

upper $\frac{1}{2}(f(2)+f(1.5)+f(1)+f(0.5))$

$\to \frac{1}{2}(9+8+7+6) \to (30)\left(\frac{1}{2}\right) \to \boxed{16}$

㉗ $g(x)=2x^2-x-1$ $[2,5]$ 6 partitions

$4x=0.5$

lower $\frac{1}{2}(f(2)+f(2.5)+f(3)+f(3.5)+f(4)+f(4.5))$

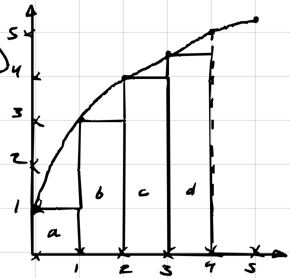
$\to \frac{1}{2}(5+9+14+20+27+35) \to \frac{1}{2}(110) \to \boxed{55}$

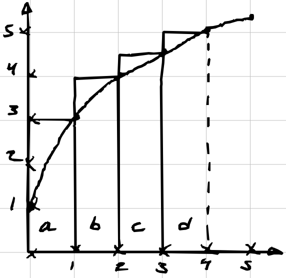
upper

$\frac{1}{2}(f(5)+f(4.5)+f(4)+f(3.5)+f(3)+f(2.5))$

$\to \frac{1}{2}(44+35+27+20+14+9) \to \frac{1}{2}(149) \to \boxed{74.5}$

㉛ ⓐ





a = 1·1 b = 3·1 c = 4·1 d = 4.5·1

→ 1+3 + 4 + 4.5 → 4+4+4.5

→ 8 + 4.5 → 12.5

$\boxed{12.5}$

a = 3·1 b = 4·1 c = 4.5 ·1

d = 5·1

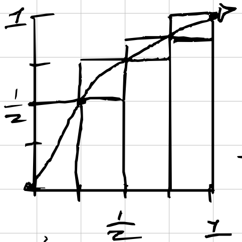
→ 3 + 4 + 4.5 + 5 → 7 + 5 + 4.5

→ 12 + 4.5 → 16.5

$\boxed{16.5}$

㉝ $y = \sqrt{x}$ [0, 1], 4 portions

$\Delta x = \frac{b-a}{n} = \frac{1-0}{4} \to \frac{1}{4}$



x	f(x
0	0
0.25	.5
0.5	.70711
0.75	.86603
1	1

Lower sum

$$s(n) = \sum_{i=1}^{n} f(m_i)\,\Delta x$$

$$\to s(n) = \sum_{i=1}^{4}\left(f(m_i)\left(\frac{1}{4}\right)\right) \to \left((0) + \left(\frac{1}{8}\right) + 0.17677 + 0.2165\right)$$

$$\to \boxed{0.51827}$$

Upper sum

$$S(n) = \sum_{i=1}^{4} f(M_i)\,\Delta x \to S(n) = \sum_{i=1}^{4} f(M_i)\frac{1}{4}$$

$$\to \frac{1}{4} + \frac{1}{8} + 0.17677 + 0.2165 \to \boxed{0.76827}$$

㊴ $$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n^3}(i-1)^2 \to \frac{n^2 x_i^2}{n^3} \to \frac{x_i^2}{n}$$

$$\to \lim_{x\to\infty} \sum_{i=1}^{n} \frac{x_i^2}{n} \to \int_0^1 x^2\,dx \text{ as } n\to\infty \to \left[\frac{x^3}{3}\right]_0^1$$

$$\to \frac{1}{3} - \frac{0}{3} \to \boxed{\frac{1}{3}}$$

㊶ $\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{i}{n}\right)\left(\frac{2}{n}\right) \to \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left(1+\frac{i}{n}\right)$

$\to 2\sum_{i=1}^{n} f(x_i)\Delta x = 2\sum_{i=1}^{n}(1+x_i)\Delta x$

$\to 2\int_0^1 (1+x)\,dx \to 2\left[x+\frac{x^2}{2}\right]_0^1 \to 2\left[\left(1+\frac{1}{2}\right)-0\right] \to 2\left(1+\frac{1}{2}\right)$

$\to 2+1 \to \boxed{3}$

㊼ $y=x^2+2$, $[0,1]$

$\Delta x = \frac{1-0}{n} \to \frac{1}{n}$

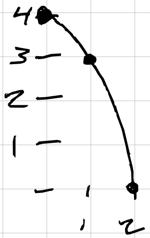
$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x \to S(n)=\sum_{i=1}^{n}\left(\left(\frac{i}{n}\right)^2+2\right)\frac{1}{n}$

$\to \sum\left(\frac{i}{n}\right)^2\left(\frac{1}{n}\right) + \sum 2\left(\frac{1}{n}\right) = 2\sum\frac{1}{n} = 2\cdot\frac{n}{n} = 2$

$\to S(n) = \left(\frac{1}{n^3}\sum_{i=1}^{n} i^2\right) + 2$

$\to \lim_{n\to\infty}\frac{1}{n^3}\sum_{i=1}^{n} i^2 = \frac{1}{3} \to \frac{1}{3}+2 = \boxed{\frac{7}{3}}$

(65) $f(x)=4-x^2$ $[0,2]$



$\boxed{6, 6}$

4.6 HW : Q. [redacted], [redacted], [redacted], [redacted]

① $\int_0^2 x^2\,dx$, $n=4$

$\to \frac{2}{8}\left(f(0)+2f(\tfrac{1}{2})+2f(1)+2f(3/2)+f(2)\right)$

$\to \frac{1}{4}\left(0+\frac{2}{1}(\frac{1}{4})+\frac{2}{1}(1)+\frac{2}{1}(\frac{9}{4})+4\right)$

$\to \frac{1}{2}(\frac{1}{4})+\frac{1}{2}(1)+\frac{1}{2}(\frac{9}{4})+1)$

$\to \frac{1}{8}+\frac{1}{2}+\frac{9}{8}+\frac{8}{8} \to \frac{10}{8}+\frac{8}{8}+\frac{4}{8} \to \frac{22}{8}$

$\to \frac{11}{4} = \boxed{2.75}$ Trap. result

Int. result

$\int_0^2 x^2\,dx \to \left[\frac{x^3}{3}\right]_0^2 \to \left(\frac{8}{3}-\frac{0}{3}\right) \to \frac{8}{3} = \boxed{2.6667}$

⑥ $\int_0^8 \sqrt[3]{x}\,dx$, $n=8$ $\quad \frac{8-0}{16} \to \frac{1}{2}$

$\to \frac{1}{2}\left(f(0)+2f(1)+2f(2)+2f(3)+2f(4)+2f(5)+2f(6)+\right.$

$\left.2f(7)+f(8)\right) \to \frac{1}{2}\left(0+2+2(2^{1/3})+2(3^{1/3})+2(4^{1/3})+\right.$

$\left.2(5^{1/3})+2(6^{1/3})+2(7^{1/3})+(8^{1/3})\right)$

Trap. Result

$\to 1+2^{1/3}+3^{1/3}+4^{1/3}+5^{1/3}+6^{1/3}+7^{1/3}+\left(\frac{(8^{1/3})}{2}\right) = \boxed{11.7296}$

$\int_0^8 \sqrt[3]{x}\,dx \to \left[\frac{3x^{4/3}}{4}\right]_0^8 \to \left(\frac{3(8^{4/3})}{4} - 0\right) = \boxed{12}$

$x^{1/3}$

Int. Result

⑨ $\int_0^1 \frac{2}{(x+2)^2}\,dx,\ n=4 \quad \left(\frac{1-0}{2(4)}\right) \rightarrow \frac{1}{8}$

$\rightarrow \frac{1}{8}\left(f(0)+2f\left(\frac{1}{4}\right)+2f\left(\frac{1}{2}\right)+2f\left(\frac{3}{4}\right)+f(1)\right)$

$\rightarrow \frac{1}{8}\left(\frac{1}{2}+\frac{2}{1}\left(\frac{32}{81}\right)+2\left(\frac{8}{25}\right)+2\left(\frac{32}{121}\right)+\frac{2}{9}\right)$

$\rightarrow \frac{1}{16}+\frac{1}{4}\left(\frac{32}{81}\right)+\frac{1}{4}\left(\frac{8}{25}\right)+\frac{1}{4}\left(\frac{32}{121}\right)+\frac{2}{72}$

$\rightarrow \boxed{0.3352}$ Trap. result

$\int_0^1 \frac{2}{(x+2)^2}\,dx \qquad u=x+2 \qquad du=dx$

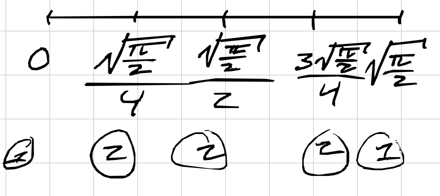
$\rightarrow \int_0^1 \frac{2}{u^2}\,du \rightarrow 2\int_0^1 u^{-2}\,du \rightarrow 2\left(\frac{u^{-1}}{-1}\right) \rightarrow -2u^{-1} \rightarrow \frac{-2}{u}$

$\rightarrow \left[\frac{-2}{x+2}\right]_0^1 \rightarrow \left(\frac{-2}{1+2}-\frac{-2}{2}\right) \rightarrow \frac{-2}{3}+1 \rightarrow \frac{1}{3}$

$\rightarrow \boxed{0.3333}$ Int. Result

⑮ $\int_0^{\sqrt{\pi/2}} \sin x^2\,dx = \frac{\sqrt{\pi/2}-0}{8}\left[f(0)+2f\left(\frac{\sqrt{\frac{\pi}{2}}}{4}\right)+2f\left(\frac{\sqrt{\frac{\pi}{2}}}{2}\right)+\right.$ $n=4$

$\left.2f\left(\frac{3\sqrt{\frac{\pi}{2}}}{4}\right)+f\left(\sqrt{\frac{\pi}{2}}\right)\right]$



$\frac{\sqrt{\frac{\pi}{2}}}{8}\left[0+0.196034+0.765367+1.546+1\right]=\boxed{0.549488}$

Trap. ~~result~~

5

Calc-found integral: $\boxed{0.5493}$

dy: * Integrating trig funcs.
↳ *Definite & indefinite
! * Integrals using "u" substitution
* Findinding limits @ infinity of sums
~~* Second fundamental theorem of Cal~~
* Average & mean integrals

Scratch for Test 4

Integrating Trig funcs

㉖ $\int(t^2-\cos t)\,dt \rightarrow \int t^2 - \int \cos t \rightarrow \frac{t^3}{3} - (\sin t) + C$

$\rightarrow \boxed{\frac{t^3}{3} - \sin(t) + C}$

㉘ $\int(\theta^2+\sec^2\theta)\,d\theta \rightarrow \int \theta^2 + \int \sec^2\theta$

$\rightarrow \boxed{\frac{\theta^3}{3} + \tan\theta + C}$

cos → sin
sin → -cos

$\int \cos x \overset{dx}{=} \sin x + C$ $\int \sin x \overset{dx}{=} -\cos x + C$

$\int \sec^2 x \overset{dx}{=} \tan x + C$ $\int \csc^2 x \overset{dx}{=} -\cot x + C$

$\int \sec x \tan x \overset{dx}{=} \sec x + C$ $\int \csc x \cot x \overset{dx}{=} -\csc x + C$

$\int \csc^2 x \, dx = -\cot x + C$

㉙ $\int(\sec^2\theta - \sin\theta)\,d\theta \rightarrow \int \sec^2\theta - \int \sin\theta$

$\rightarrow \boxed{\tan\theta + \cos\theta + C}$

① $\int \cos^2 x \sin x \, dx$ $u = \cos x$ $du = -\sin x \, dx$

$\rightarrow -\int u^2 \, du \rightarrow -\left[\frac{u^3}{3}\right] + C \rightarrow \boxed{-\frac{\cos x^3}{3} + C}$

② $\int_0^{\pi/6} \sec^2 x\,dx \to [\tan x]_0^{\pi/6}$ $\frac{\sin x}{\cos x}$

$\to [(\frac{1/2}{2/\sqrt{3}}) - (\frac{0}{1})] \to [(\frac{1}{2} \cdot \frac{2}{\sqrt{3}}) - 0] \to \boxed{\frac{1}{\sqrt{3}}}$

③ $\int (\frac{x^2 - x}{\sqrt{x}})\,dx \to \int (\frac{x^2}{\sqrt{x}})\,dx - \int (\frac{x}{\sqrt{x}})\,dx$

$\to \int x^{3/2}\,dx - \int x^{1/2}\,dx \to \frac{2x^{5/2}}{5} - \frac{2x^{3/2}}{3} + C$

$\to \boxed{\frac{2}{5} x^{5/2} - \frac{2}{3} x^{3/2} + C}$

u-Substitution

④ $\int 3x(x^2+7)^3\,dx \to$ $u = x^2 + 7$ $du = 2x\,dx$

$\to \frac{3}{2} \int u^3\,du \to \frac{3}{2}[\frac{u^4}{4}] + C \to \frac{3}{2}[\frac{(x^2+7)^4}{4}] + C$

$\to \boxed{\frac{3(x^2+7)^4}{8} + C}$

⑫ $\int \frac{2x}{\sqrt{x+1}}\,dx \to$ $u = x+1 \Rightarrow x = u - 1$

$du = dx$

$\to 2 \int \frac{x}{\sqrt{x+1}}\,dx \to 2 \int \frac{u-1}{\sqrt{u}}\,du \to 2 \int u^{1/2} - u^{1/2}\,du$

$\to 2(\frac{2u^{3/2}}{3} - 2u^{1/2}) + C \to \boxed{\frac{4}{3}(x+1)^{3/2} - 4(x+1)^{1/2} + C}$

FTC II

⑦ $\frac{d}{dx} \int_1^{2x^3} (t-3)^4\,dt \Big| \to \boxed{(2x^3 - 3)^4 (6x^2)}$

(81) $\frac{d}{dx} \int_{-2}^{x} (t^2 - 2t)\,dt \to x^2 - 2x$

(85) $\frac{d}{dx} \int_0^{x} t \cos t\,dx \to x \cos x$

(91) $\frac{d}{dx} \int^{x^3} \sin t^2\,dt \to (\sin x^6)(3x^2)$

(86) $\frac{d}{dx}\int_0^{\sin x} \sqrt{t}\, dt \rightarrow (\sqrt{\sin x})(\cos x)$

(87) $\frac{d}{dx}\int_x^{x+2} (4t+1)\, dt \rightarrow ((4(x+2)+1)(1)) - ((4x+1)(1))$

$\rightarrow 4x+8+1-4x-1 \rightarrow \boxed{8}$

(88) $\frac{d}{dx}\int_{-x}^{x} t^3\, dt \rightarrow ((x^3)(1)) - ((-x^3)(-1))$

$\rightarrow x^3 - x^3 \rightarrow \boxed{0}$

(90) $\frac{d}{dx}\int_2^{x^2} \frac{1}{t^3}\, dt \rightarrow \boxed{\left(\frac{1}{x^6}\right)(2x)}$

(92) $\frac{d}{dx}\int_0^{x^2} \sin\theta^2\, d\theta \rightarrow \boxed{(\sin x^4)(2x)}$

Lims @ ∞ of Sums & Sums

(6) $s(n) = \sum_{i=1}^{n} \left(1+\frac{i}{n}\right)^2 \frac{2}{n}$ $\left(\lim_{n\to\infty} s(n)\right)$

$\rightarrow s(n) = \sum_{i=1}^{n} f(m_i)\Delta x \rightarrow s(n) = \sum_{i=1}^{n}\left(1+\frac{i}{n}\right)^2\left(\frac{2}{n}\right)$

$\rightarrow \frac{2}{n}\sum_{i=1}^{n}\left(1+\frac{2i}{n}+\frac{i^2}{n^2}\right) \rightarrow \frac{2}{n}\left(\sum_{i=1}^{n} 1 + \frac{2}{n}\sum_{i=1}^{n} i + \frac{1}{n^2}\sum_{i=1}^{n} i^2\right)$

$\rightarrow \frac{2}{n}\left(\left(n\cdot\frac{6n}{6n}\right)+\left(\frac{3}{2}\cdot\frac{2}{n}\left(\frac{n(n+1)}{2}\right)\right)\right)+\frac{1}{\cancel{n}^2_n}\left(\frac{\cancel{n}(n+1)(2n+1)}{6}\right)\Bigg)$

$\rightarrow \frac{\overset{1}{\cancel{2}}}{n}\left(\frac{6n^2+6n^2+6n+2n^2+3n+1}{3\cancel{6}n}\right)$

$\rightarrow \frac{14n^2+9n+1}{3n^2}$

equal exponents
→ coefficients

$\lim_{n\to\infty} s(n) = \boxed{\frac{14}{3}}$

(6) $s(n) = \sum_{i=1}^{n}\left(1+\frac{i}{n}\right)^2\left(\frac{2}{n}\right)$ find lim. of s(n) as n → ∞

$\rightarrow s(n) = \sum_{i=1}^{n} f(m_i)\Delta x \rightarrow s(n) = \sum_{i=1}^{n}\left(1+\frac{i}{n}\right)^2\left(\frac{2}{n}\right)$

$\rightarrow \frac{2}{n}\sum_{i=1}^{n}\left(1+\frac{2i}{n}+\frac{i^2}{n^2}\right) \rightarrow \frac{2}{n}\left(\sum_{i=1}^{n} 1 + \frac{2}{n}\sum_{i=1}^{n} i + \frac{1}{n^2}\sum_{i=1}^{n} i^2\right)$

$= \frac{2}{n}\left(n\cdot\frac{6n}{6n} + \frac{3}{2}\cdot\frac{2}{n}\left(\frac{n(n+1)}{2}\right) + \frac{1}{\cancel{n^2}\,n}\left(\frac{\cancel{n}(n+1)(2n+1)}{6}\right)\right)$

$\rightarrow \frac{\overset{1}{\cancel{2}}}{n}\left(\frac{6n^2+6n^2+6n+2n^2+3n+1}{3\cancel{6}n}\right)$

$= \frac{14n^2+9n+1}{3n^2} \rightarrow \lim_{n\to\infty} s(n) = \boxed{\frac{14}{3}}$

$$① \sum_{i=1}^{6}(3i+2) \rightarrow (3(1)+2)+(3(2)+2)+(3(3)+2)+(3(4)+2)$$

$$+(3(5)+2)+(3(6)+2) \rightarrow 5+8+11+14+17+20$$

$$\rightarrow 5+11+14+45 \rightarrow 11+14+50 \rightarrow 25+50 \rightarrow \boxed{75}$$

$$3\sum_{i=1}^{6} i + \sum_{i=1}^{6} 2 \rightarrow 3\left(\frac{6(6+1)}{2}\right)+12$$

$$\rightarrow 3\left(\frac{42}{2}\right)+12 \rightarrow 3(21)+12 \rightarrow 63+12 \rightarrow \boxed{75}$$

$$③ \sum_{k=0}^{4}\frac{1}{k^2+1}$$

muos

$$① \sum_{i=1}^{n} c = cn, \text{ c is a constant}$$

$$② \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad ③ \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$④ \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\rightarrow \left(\frac{1}{0^2+1}\right)+\left(\frac{1}{1^2+1}\right)$$

$$+\left(\frac{1}{2^2+1}\right)+\left(\frac{1}{3^2+1}\right)$$

$$+\left(\frac{1}{4^2+1}\right) \rightarrow 1+\frac{1}{2}+\frac{1}{5}+\frac{1}{10}+\frac{1}{17}=\frac{158}{85}$$

$$⑤ \sum_{k=1}^{4} c \rightarrow 4c \qquad ⑦ \sum_{i=1}^{11}\frac{1}{5i}$$

$$⑮ \sum_{i=1}^{24} 4i \rightarrow 4\left(\frac{n(n+1)}{2}\right) \rightarrow 4\left(\frac{24(24+1)}{2}\right) \rightarrow 1200$$

$$⑰ \sum_{i=1}^{20}(i-1)^2 \rightarrow \sum_{i=1}^{20} i^2-2i+1 \rightarrow \sum_{i=1}^{20} i^2 - 2\sum_{i=1}^{20} i + \sum_{i=1}^{20} 1$$

$$(i-1)$$

$$\xrightarrow{n=20}\left(\frac{n(n+1)(2n+1)}{6}+-2\left(\frac{n(n+1)}{2}\right)+20\right) \rightarrow \frac{20(21)(41)}{6}-2\left(\frac{20(21)}{2}\right)$$

$$+20 \rightarrow (2870-420+20) \rightarrow \boxed{2470}$$

$$⑲ \sum_{i=1}^{15}(i(i-1)^2) \rightarrow \sum_{i=1}^{15}\left[i(i^2-2i+1)\right] \rightarrow \sum_{i=1}^{15} i^3-2i^2+i \rightarrow \sum_{i=1}^{15}\left(\frac{n^2(n+1)^2}{4}\right)$$

$$-2\sum_{i=1}^{15}\frac{n(n+1)(2n+1)}{6}+\sum_{i=1}^{15}\frac{n(n+1)}{2} \rightarrow \frac{15^2(16^2)}{4}-2\left(\frac{15(16)(31)}{6}\right)+\frac{15(16)}{2}$$

$$\rightarrow 14400-2480+120 \rightarrow \boxed{12040}$$

$\boxed{37}$ $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{24i}{n^2} \to \frac{24}{n^2} \sum_{i=1}^{n} i \to \frac{24}{n^2}\left(\frac{n(n+1)}{2_n}\right)$

$\to \frac{24(n^2+n)}{2n^2} \to \frac{24n^2+24n}{2n^2} \to \frac{12\cancel{n}^{2}+12\cancel{n}}{\cancel{n}^{2}n}$

$\to \frac{12n+12}{n}$

$\to \boxed{12}$

$\boxed{39}$ $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n^3}(i-1)^2 \to \frac{1}{n^3}\left(\sum_{i=1}^{n} i^2 - 2i + 1\right)$

$\to \frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6} - 2\left(\frac{n(n+1)}{2}\right) + n\right)$

$\to \frac{1}{n^3}\left(\frac{n(2n^2+n+2n+1)}{6} - 2\left(\frac{n^2+n}{2}\right) + \frac{n}{1}\right)$

$\to \frac{1}{n^3}\left(\left(\frac{2n^3+3n^2+n}{6}\right) - (n^2+2) + \frac{n}{1}\right)$

$\to \frac{1}{n^3}\left(\left(\frac{2n^3+3n^2+n}{6}\right) - \frac{6n^2+12}{6} + \frac{6n}{6}\right)$

$\to \frac{1}{n^3}\left(\frac{2n^3+3n^2+7n+12}{6}\right) \to \frac{2n^3+3n^2+7n+12}{6n^3}$

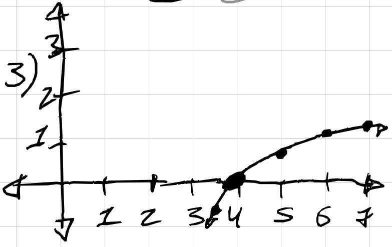
$\to \frac{2}{6} \to \boxed{\frac{1}{3}}$

5.1 HW p 325: (~~1~~, ~~5-~~8, ~~13~~, ~~21~~, ~~23~~, ~~24~~, ~~30~~, ~~33~~, ~~43~~, ~~45~~, ~~49~~, ~~51~~, ~~57~~)

① $\ln 45 \rightarrow \boxed{3.8067}$ $\qquad \int_1^{45} \frac{1}{x}\,dx \rightarrow \boxed{3.8067}$

⑤ $\boxed{b}$ ⑥ $\boxed{d}$ ⑦ $\boxed{a}$ ⑧ $\boxed{c}$

⑬ $f(x) = \ln(x-3)$



domain: $(3, \infty)$

㉑

$\ln \frac{xy}{z} \rightarrow \ln(xy) - \ln(z) \rightarrow \boxed{(\ln(x) + \ln(y)) - \ln(z)}$

㉓ $\ln(x\sqrt{x^2+5}) \rightarrow \ln x + \ln\sqrt{x^2+5}$

$\rightarrow \boxed{\ln x + \frac{1}{2}(\ln x^2+5)}$

㉔ $\ln\sqrt{a-1} \rightarrow \boxed{\frac{1}{2}\ln(a-1)}$

㉚ $3\ln x + 2\ln y - 4\ln z \rightarrow \ln x^3y^2 - 4\ln z$

$\rightarrow \boxed{\ln \frac{x^3y^2}{z}}$

㉝ $2\ln 3 - \frac{1}{2}\ln(x^2+1) \rightarrow \ln 9 - \ln\sqrt{x^2+1}$

$\rightarrow \boxed{\ln \frac{9}{\sqrt{x^2+1}}}$

㊸ $g(x) = \ln x^2 \rightarrow g'(x) = \frac{2x}{x^2} \rightarrow \boxed{g'(x) = \frac{2}{x}}$

㊺ $y = (\ln x)^4 \rightarrow y' = 4(\ln x)^3(\frac{1}{x}) \rightarrow \boxed{y' = \frac{4(\ln x)^3}{x}}$

(49) $y=\ln(x\sqrt{x^2-1}) \rightarrow \ln x+\ln\sqrt{x^2-1}$

$\rightarrow \frac{d}{dx}\left(\ln x+\frac{1}{2}(\ln(x^2-1))\right) \rightarrow \frac{1}{x}+\frac{1}{2}\left(\frac{2x}{x^2-1}\right)$

$\rightarrow \frac{1}{x}+\frac{2x}{\cancel{2}(x^2-1)} \rightarrow \frac{1}{x}+\frac{x}{x^2-1} \rightarrow \frac{x^2-1}{x^3-x}+\frac{x^2}{x^3-x}$

$\rightarrow \frac{x^2+x^2-1}{x^3-x} \rightarrow \frac{2x^2-1}{x^3-x} \rightarrow \boxed{y'=\frac{2x^2-1}{x(x^2-1)}}$

(51) $f(x)=\ln\left(\frac{x}{x^2+1}\right) \rightarrow \ln x-\ln(x^2+1)$

$\rightarrow \frac{1}{x}-\left(\frac{2x}{x^2+1}\right) \rightarrow \frac{x^2+1}{x^3+x}-\frac{2x^2}{x^3+x} \rightarrow \frac{x^2-2x^2+1}{x^3+x}$

$\rightarrow \frac{-x^2+1}{x^3+x} \rightarrow \boxed{f'(x)=\frac{-x^2+1}{x(x^2+1)}}$

(57) $y=\ln\sqrt{\frac{x+1}{x-1}} \rightarrow \frac{1}{2}\ln\frac{x+1}{x-1} \rightarrow \frac{1}{2}\ln(x+1)-\frac{1}{2}\ln(x-1)$

$\rightarrow \frac{1}{2}\left(\frac{1}{x+1}\right)-\frac{1}{2}\left(\frac{1}{x-1}\right) \rightarrow \frac{1}{2x+2}-\frac{1}{2x-2} \rightarrow$

$\rightarrow \frac{2x-2}{4x^2-4}-\frac{2x+2}{4x^2-4} \rightarrow \frac{-4}{4x^2-4} \rightarrow \frac{-1}{x^2-1}$

$\rightarrow \boxed{y'=\frac{1}{1-x^2}}$

5.2 HW

~~1-7~~ odd, ~~17~~, [illegible] odd, 2~~7~~, ~~37~~, ~~41~~, [illegible], ~~51~~, [illegible]

(43, omit x from 9)

① $\int \frac{5}{x}\,dx \rightarrow$ $u = x$ $du = 1\,dx$

$\rightarrow 5\int \frac{1}{x}\,dx \rightarrow \boxed{5(\ln|x|)+C}$

③ $\int \frac{1}{x+1}\,dx \rightarrow \int \frac{1}{u}\,du$ $u = x+1$ $du = 1\,dx$

$\rightarrow \ln|u| + C \rightarrow \boxed{\ln|x+1|+C}$

⑤ $\int \frac{1}{2x+5}\,dx$ $u = 2x+5$ $du = 2\,du$

$\rightarrow \frac{1}{2}\int \frac{1}{u}\,du \rightarrow \frac{1}{2}\ln|u| + C \rightarrow \boxed{\frac{1}{2}\ln|2x+5|+C}$

⑦ $\int \frac{x}{x^2-3} \rightarrow u = x^2-3$ $du = 2x\,dx$

$\rightarrow \frac{1}{2}\int \frac{1}{u}\,du \rightarrow \frac{1}{2}\ln|u| + C \rightarrow \boxed{\frac{1}{2}\ln|x^2-3|+C}$

⑨ $\int \frac{4x^3+3}{x^4+3x}\,dx$ $u = x^4+3x$ $du = 4x^3+3\,dx$

$\rightarrow \int \frac{1}{u}\,du \rightarrow \ln|u| + C \rightarrow \boxed{\ln|x^4+3x| + C}$

⑰ $\int \frac{x^3-3x^2+5}{x-3} \rightarrow u = x-3$ $du = dx$

$x = u+3$

$$\rightarrow \begin{array}{r} x^2 + \frac{5}{x-3} \\ x-3\,\overline{)\,x^3-3x^2+5} \\ \underline{-\;x^3-3x^2+} \\ \textcircled{5}+ \end{array} \rightarrow \int x^2 + \frac{5}{x-3}$$

$u = x-3$
$du = dx$

$\rightarrow \int x^2 + \int \frac{5}{x-3}$

$\rightarrow \frac{x^3}{3} + 5\int \frac{1}{u}\,dx + C$

$\rightarrow \boxed{\frac{x^3}{3} + 5\ln|x-3| + C}$

(21) $\int \frac{(\ln x)^2}{x}\,dx \qquad u = \ln x \qquad du = \frac{1}{x}\,dx$

$\int u^2\,du \rightarrow \frac{u^3}{3} + C \rightarrow \boxed{\frac{(\ln x)^3}{3} + C}$

(23) $\frac{2}{3}\int_2^3 \frac{1}{2\sqrt{x}\,(1-3\sqrt{x})}\,dx \rightarrow u = 1-3\sqrt{x} \qquad du = \frac{-3}{2\sqrt{x}}$

$\rightarrow \frac{2}{3}\int \frac{1}{u}\,du \rightarrow -\frac{2}{3}\ln|u| + C \rightarrow \boxed{-\frac{2}{3}\ln|1-3\sqrt{x}| + C}$

(25) $\int \frac{2x}{(x-1)^2}\,dx \rightarrow \int \frac{2x+2-2}{(x-1)^2}\,dx$

$\rightarrow 2\int \frac{x-1}{(x-1)^2}\,dx + 2\int \frac{1}{(x-1)^2}\,dx \rightarrow$

$2\int \frac{1}{x-1}\,dx + 2\int \frac{1}{(x-1)^2}\,dx \rightarrow \boxed{2\ln|x-1| - \frac{2}{(x-1)} + C}$

$2\int (x-1)^{-2}\,dx \rightarrow \begin{matrix} u = x-1 \\ du = dx \end{matrix} \rightarrow 2\int u^{-2}\,du \rightarrow 2\frac{u^{-1}}{-1} + C \rightarrow \frac{-2}{(x-1)} + C$

(27) $\int \frac{1}{1+\sqrt{2x}}\,dx \qquad u = 1+\sqrt{2x} \qquad du = \frac{1}{\sqrt{2x}}\,dx$

$(2x)^{1/2}$

$2x\,du = dx \rightarrow (u-1)\,du = dx$

$\rightarrow \int \frac{1}{1+\sqrt{2x}}\,dx \rightarrow \int \frac{(u-1)}{u}\,du \rightarrow \int \left(1 - \frac{1}{u}\right)du$

$\rightarrow u - \ln|u| + C_1 \rightarrow \boxed{\sqrt{2x} - \ln|1+\sqrt{2x}| + C}$

(37) $\int \frac{\cos t}{1+\sin t}\,dt \rightarrow u = 1+\sin t \qquad du = \cos t\,dt$

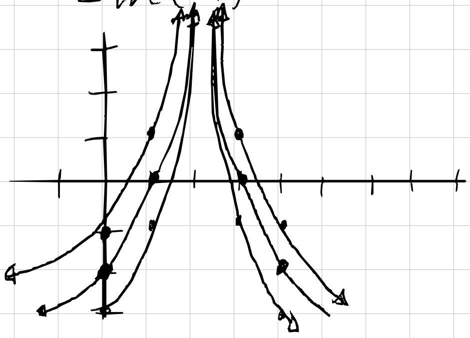
$\rightarrow \int \frac{1}{u}\,du \rightarrow \ln|u| + C \rightarrow \boxed{\ln|1+\sin t| + C}$

㊶ $\frac{dy}{dx} = \frac{3}{2-x}$, $(1,0) \to -3\int \frac{1}{x-2}\,dx$

$\to u = x-2 \quad du = dx \to -3\int \frac{1}{u}\,du \to \boxed{-3\ln|x-2|+C}$

$\to 0 = -3\ln|1-2| + C \to 0 = -3\ln(+1) + C$

$\to 0 = -3(0) + C \to C = 0$



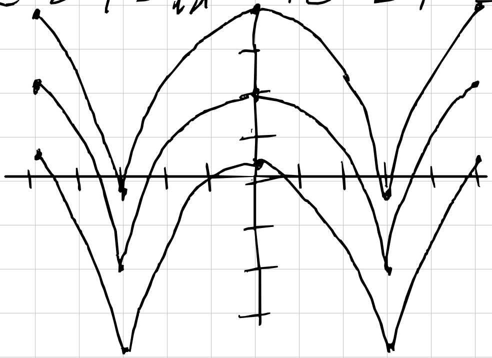
㊸ $\frac{dy}{dx} = \frac{2x}{x^2-9}$, $(0,4)$

$\to u = x^2-9 \quad du = 2x \quad dx$

$\to \boxed{\ln|x^2-9|+C}$

$C \to 4 = \ln|0^2-9| + C \to 4 = \ln 9 + C \to 4 = 2.197 + C$

$\to \underline{C = 1.803}$



(51) $\int_1^e \frac{(1+\ln x)^2}{x}\,dx$

$\to u = 1+\ln x$

$du = \frac{1}{x}\,dx$

$\to \int_1^e u^2\,du \to \left[\frac{(1+\ln x)^3}{3}\right]_1^e \to \left[\frac{(1+1)^3}{3} - \frac{(1+0)^3}{3}\right]$

$\to \frac{8}{3} - \frac{1}{3} \to \boxed{\frac{7}{3}}$

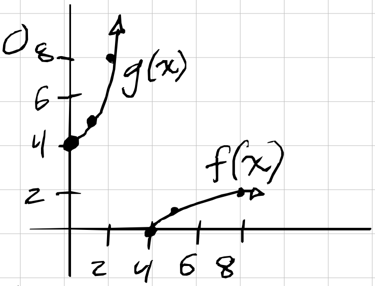
(65) $F(x) = \int_1^{3x} \frac{1}{t}\,dt \to \left(\frac{1}{3x}\right)(3) \to \boxed{F'(x) = \frac{1}{x}}$

5.3 HW p.343: (~~5~~, ~~13~~, ~~15~~, [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible])

⑤ $f(x)=\sqrt{x-4}$ $\quad g(x)=x^2+4,\ x\geq 0$

$\sqrt{x^2+4-4} \to \sqrt{x^2} \to x$

$\sqrt{x-4}^{\,2}+4 \to x-4+4 \to x$



⑬ $f(x)=\frac{3}{4}x+6$ → passes the horizontal line test,

$\boxed{\text{there is an inverse function}}$

⑮ $f(\theta)=\sin\theta$ → does not pass the horizontal line test

$\boxed{\text{no inverse function}}$

⑰ $h(s)=\frac{1}{s-2}-3$ → passes the horizontal line test

$\boxed{\text{there is an inverse}}$

㉕ $f(x)=\frac{x^4}{4}-2x^2 \to \boxed{f'(x)=x^3-4x}$

→ The function isn't monotonic, $\boxed{\text{no inverse}}$

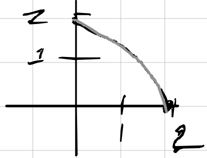
㉗ $f(x)=\ln(x-3) \to f'(x)=\frac{1}{x-3}$ → monotonic, $\boxed{\text{inverse exists}}$

㊶ $f(x)=\sqrt{4-x^2}\quad 0\leq x\leq 2$

@ $x^2=\sqrt{4-y^2}^{\,2} \to x^2=4-y^2 \to x^2+y^2=4 \to y^2=4-x^2$

$\to y=\sqrt{4-x^2} \to \boxed{f^{-1}(x)=\sqrt{4-x^2},\ 0\leq x\leq 2}$

ⓑ



ⓒ They appear to occupy the same space because the origin reflection is perfectly centered between them and they look the same

ⓓ both have a domain of $[0, 2)$ and range of $(0, 2)$

(49) ⓐ Because x represents lbs of the commodity costing 1.25 / lbs and the ~~total~~ is 50 lbs, $(x-50)$ represents lbs of the other commodity ~~so~~ that the total pounds always add up to 50. $y = 1.25 + 1.6(50-x)$ represents the total cost at 50 lbs total depending on the lbs of the commodity costing 1.25.

ⓑ $y = 1.25 + 1.6(50-x) \rightarrow y = -0.35x + 80$

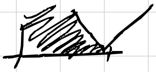
$\rightarrow x = -0.35y + 80 \rightarrow 0.35y = 80 - x \rightarrow y = \frac{(80-x)}{0.35}$

x represents cost & y represents lbs

ⓒ $\boxed{\text{domain: } (62.5, 80)}$

ⓓ $73 = -0.35x + 80 \rightarrow -7 = -0.35x \rightarrow \boxed{20 \text{ lbs}}$

(53) $f(x) = |x-2|,\ x \leq 2 \rightarrow$



one to one ✓

$= -(x-2) \rightarrow -x+2 \rightarrow 2-x = y \rightarrow 2-y = x$

$\rightarrow f^{-1}(x) = 2-x,\ x \geq 0$

(59) function is one to one, there is an inverse which represents time given a volume of water.

(63) $f(x)=5-2x^3,\ a=7 \rightarrow f'(x)=-6x^2$

$x=5-2y^3 \rightarrow 2y^3=5-x \rightarrow y^3=\frac{5-x}{2} \rightarrow y=\sqrt[3]{\frac{5-x}{2}}$

$(f^{-1})'(7)=\frac{1}{f'(f^{-1}(7))}=\frac{1}{f'(-1)} \rightarrow \frac{1}{-6(-1)^2} \rightarrow \frac{1}{-6(1)} \rightarrow \boxed{\frac{1}{-6}}$

(67) $f(x)=\sin(x),\ -\frac{\pi}{2} \le x \le \frac{\pi}{2},\ a=\frac{1}{2}$

$f'(x)=\cos(x) \rightarrow f^{-1}(x)=\sin^{-1}(x)$ $\quad f\left(\frac{\pi}{6}\right)=\frac{1}{2}$

$f^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

$\rightarrow (f^{-1})'\left(\frac{1}{2}\right) \rightarrow \frac{1}{f'(f^{-1}(\frac{1}{2}))} \rightarrow \frac{1}{f'\left(\frac{\pi}{6}\right)}$

$\rightarrow \frac{1}{\cos\frac{\pi}{6}} \rightarrow \frac{1}{\frac{\sqrt{3}}{2}} \rightarrow \boxed{\frac{2}{\sqrt{3}}}$

(69) $f(x)=\frac{x+6}{x-2},\ x>2,\ a=3$ $\quad f(6)=3$

$f^{-1}(x) \rightarrow x=\frac{y+6}{y-2} \rightarrow (y-2)(x)=y+6$ $\quad f^{-1}(3)=6$

$\rightarrow xy-2x=y+6 \rightarrow -2x=y-xy+6$

$\rightarrow f^{-1}(x)=\frac{2x+6}{x-1}$ $\quad f'(x)=\frac{(x-2(1))-(x+6)(1)}{(x-2)^2}$

$\rightarrow f'(x)=\frac{x-2-x-6}{(x-2)^2}=\frac{-8}{(x-2)^2}$

$(f^{-1})'(3)=\frac{1}{f'(f^{-1}(3))} \rightarrow \frac{1}{f'(6)} \rightarrow \frac{1}{-0.5} \rightarrow \frac{1}{1} \cdot -\frac{2}{1} \rightarrow \boxed{-2}$

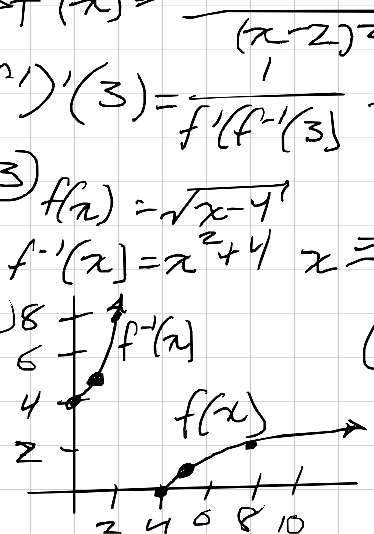
(73) $f(x)=\sqrt{x-4}$

$f^{-1}(x)=x^2+4 \quad x \ge 0$ $\quad \frac{(5,1)}{(1,5)}$

ⓐ $f(x)\ (4,\infty),\ f^{-1}(x) \rightarrow (0,\infty)$

ⓑ $f(x)\ (0,\infty),\ f^{-1}(x)\ (4,\infty)$

ⓒ



ⓓ $f'(x)=\frac{1}{2\sqrt{x-4}}$ $\quad f'(5)=\boxed{\frac{1}{2}}$

$(f^{-1})'(x)=2x$ $\quad (f^{-1})'(1)=\boxed{2}$

5.4 HW: [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted]

② $e^{\ln 3x} = 24 \to \ln 24 = (\ln 3x)(\ln e)$

$\to \ln 24 = \ln 3x \to \ln 24 = \ln 3 + \ln x$

$\to \ln 24 - \ln 3 = \ln x \to x = e^{(\ln 24 - \ln 3)} = \boxed{8}$

③ $e^{x} = 12 \to x = \ln 12 \to \boxed{x = 2.485}$

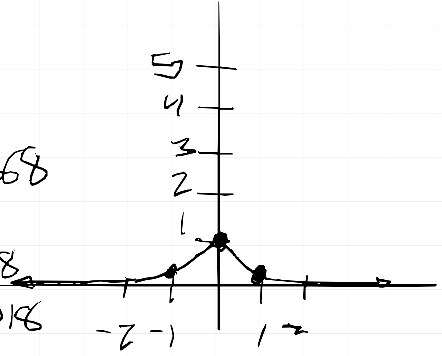
⑥ $8e^{x} - 12 = 7 \to 8e^{x} = 19 \to e^{x} = \frac{19}{8} \to x = \ln \frac{19}{8} = \boxed{0.865}$

⑪ $\ln x = 2 \to x = e^{2} \to \boxed{x = 7.389}$

⑭ $\ln 4x = 1 \to 4x = e \to x = \frac{e}{4} \to \boxed{x = 0.68}$

㉑ $y = e^{-(x^2)}$

x	y
-1	$\frac{1}{e} \approx 0.368$
0	1
1	$\frac{1}{e} \approx 0.368$
2	$\frac{1}{e^4} \approx 0.018$



㉕ c ㉖ d ㉗ a ㉘ b

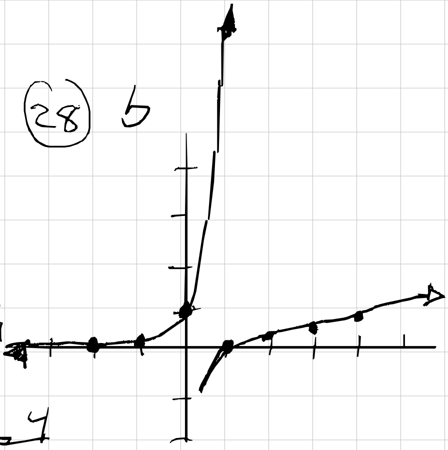
㉙ $f(x) = e^{2x}$ $g(x) = \ln\sqrt{x}$

$f(x)$:

x	-2	-1	0	1	2	3	4
y	.02	.13	1	7.3	54	403	2981

$g(x)$:

x	-2	-1	0	1	2	3	4
g				0	.3	.5	.7



(39) $y=e^x\ln x \rightarrow e^x\left(\frac{1}{x}\right)+e^x(\ln x) \rightarrow \boxed{e^x\left(\frac{1}{x}+\ln x\right)}$

(41) $y=x^3e^x \rightarrow 3x^2e^x+x^3e^x \rightarrow \boxed{e^x(x^3+3x^2)}$

(44) $g(t)=e^{-3/t^2} \rightarrow \boxed{e^{-3/t^2}\left(\frac{6}{t^3}\right)}$ $\quad -3t^{-2} \rightarrow \frac{6}{t^3}$

(47) $y=\frac{2}{e^x+e^{-x}} \rightarrow u=e^x+e^{-x}$

$y=2\cdot\left[\frac{u'}{u^2}\right] \rightarrow 2\left[\frac{e^x-e^{-x}}{(e^x+e^{-x})^2}\right] \rightarrow \boxed{\frac{2(e^x-e^{-x})}{(e^x+e^{-x})^2}}$

(49) $y=\frac{e^x+1}{e^x-1} \rightarrow \frac{((e^x-1)(e^x))-((e^x+1)(e^x))}{(e^x-1)^2}$

$\rightarrow \frac{e^x((e^x-1)-(e^x+1))}{(e^x-1)^2} \rightarrow \frac{e^x(-2)}{''} \rightarrow \boxed{\frac{-2e^x}{(e^x-1)^2}}$

(56) $f(x)=e^{-2x}$, $(0,1) \rightarrow f'(x)=(e^{-2x})-2 \rightarrow \boxed{-2e^{-2x}}$

$\rightarrow -2e^{-2(0)} \rightarrow -2(e^0) \rightarrow -2(1) \rightarrow -2=f'(0)$

$\rightarrow y-1=-2(x-0) \rightarrow \boxed{y=-2x+1}$

(59) $f(x)=e^{-x}\ln x \rightarrow f'(x)=e^{-x}\left(\frac{1}{x}\right)-e^{-x}(\ln x)$

$(1,0)$

$f'(1)=e^{-1}\left(\frac{1}{1}\right)-e^{-1}(\ln(1)) \rightarrow \frac{1}{e}-\frac{\ln(1)}{e} \rightarrow \frac{1}{e}-\frac{0}{e} \rightarrow \boxed{\frac{1}{e}}$

$\rightarrow y-0=\frac{1}{e}(x-1) \rightarrow \boxed{y=\frac{1}{e}x-\frac{1}{e}}$

(67) $f(x)=(3+2x)e^{-3x} \rightarrow 3e^{-3x}+2xe^{-3x} \rightarrow -9e^{-3x}+(2x(-3e^{-3x})+2e^{-3x})$

$\rightarrow f'(x)=-9e^{-3x}-6xe^{-3x}+2e^{-3x} \rightarrow -7e^{-3x}-6xe^{-3x} \rightarrow (-7-6x)e^{-3x}$

$\rightarrow f''(x) \rightarrow (-7-6x)(-3e^{-3x})+(-6)(e^{-3x}) \rightarrow 21e^{-3x}+18xe^{-3x}-6e^{-3x}$

$\rightarrow 15e^{-3x}+18xe^{-3x} \rightarrow (15+18x)e^{-3x} \rightarrow \boxed{3(5+6x)e^{-3x}=f''(x)}$

5.5 HW [~~3,5~~, ~~15-18~~, ~~23,24~~, ~~35~~, [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], ~~71~~, [illegible], [illegible], [illegible]]

③ $\frac{\ln 1}{\ln 7} \to \frac{0}{\ln 7} \to$ ⓪

⑤ ⓐ $2^3=8$ $\boxed{\log_2 8=3}$ ⓑ $3^{-1}=\frac{1}{3} \to \boxed{\log_3 \frac{1}{3}=-1}$

⑮ d ⑯ c ⑰ b ⑱ a

㉓ ⓐ $x^2-x=\log_5 25 \to x^2-x-\log_5 25=0$

$\to x^2-x-\left(\frac{\ln 25}{\ln 5}\right) \to x^2-x-2=0 \to (x+1)(x-2)=0$

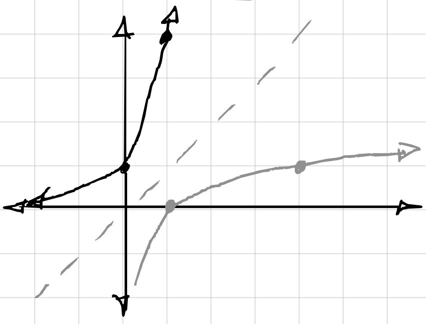
$\boxed{x=-1,2}$

ⓑ $3x+5=\log_2 64$

$\to 3x+5=\log_2 2^6 \to 3x+5=6 \to 3x=1 \to \boxed{x=\frac{1}{3}}$

㉔ ⓐ $\log_3 x+\log_3(x-2)=1 \to 3^1=x-2 \to \boxed{x=1}$

㉟ $f(x)=4^x$ $g(x)=\log_4 x$



㊴ $y=5^{-4x} \to 5^u$

$\to 5^u \ln 5\ u' \to 5^{-4x}(\ln 5)-4$

$\to \boxed{\frac{-4\ln 5}{625^x}}$

㊶ $f(x)=x\overset{g(x)}{\boxed{9^x}} \to 1(g(x))+x(g'(x)) \to g(x)=9^x \to 9^x(\ln 9)$

$\to 1(9^x)+x(9^x(\ln 9)) \to \boxed{9^x(1+x\ln 9)}$

㊸ $g(t)=t^2 2^t \to t^2(u')+2t(u)$ $g(x): 2^t$ $g'(x)=2^t \ln 2$

$\to t^2(2^t \ln 2)+2t(2^t) \to \boxed{2^t t(2+t\ln 2)}$

(44) $f(t)=\frac{3^{2t} \leftarrow 1}{t \leftarrow 2} \rightarrow \quad 1' = 3^{2t}(\ln 3) \quad 2' = 1$

$\rightarrow \frac{t(3^{2t}(\ln 3)(2)) - 3^{2t}(1)}{t^2} \rightarrow \boxed{\frac{3^{2t}(2t(\ln 3)-1)}{t^2}}$

(49) $h(t) = \log_5 (4-t)^2 \longrightarrow \left(\frac{\ln(4-t)^2}{\ln 5}\right)$

$\rightarrow \frac{1}{(4-t)^2} \cdot 2(4-t)(-1) \rightarrow \frac{2(4-t)(-1)}{(4-t)^2} \rightarrow \frac{-2}{4-t}$

$\frac{-2}{4-t} \cdot \frac{1}{\ln 5} \rightarrow \boxed{\frac{-2}{\ln 5(4-t)}}$

(57) $g(t) = \frac{10 \log_4 t}{t} \rightarrow \frac{10}{\ln 4}\left(\frac{\ln t}{t}\right) \rightarrow \frac{10}{\ln 4}\left(\frac{t(\frac{1}{t}) - \ln t(1)}{t^2}\right)$

$\rightarrow \frac{10}{\ln 4}\left(\frac{1-\ln t}{t^2}\right) \rightarrow \frac{10-10\ln t}{t^2 \ln 4} \rightarrow \boxed{\frac{5(1-\ln t)}{t^2 \ln 2}}$

(59) $y = 2^{-x}, (-1,2) \rightarrow \ln y = \ln 2^{-x} \rightarrow \ln y = -x \ln 2$

$\rightarrow \frac{y'}{y} = -\ln 2 \rightarrow y' = y(-\ln 2) \rightarrow -2^{-x} \ln 2$

$y'\big|_{(-1,2)} = -2^{-1} \ln 2 \rightarrow -2 \ln 2 \rightarrow y-2 = -2\ln 2(x+1)$

$\rightarrow \boxed{y = -2(\ln 2)x - (2\ln 2) + 2}$

(61) $y = \log_3 x \quad (27,3) \rightarrow \frac{\ln x}{\ln 3} \rightarrow \frac{1}{\ln 3} \cdot (\ln x) \rightarrow \left(\frac{1}{\ln 3} \cdot \frac{1}{x}\right)$

$\rightarrow \frac{1}{x \ln 3} \quad y'\big|_{(27,3)} = \frac{1}{27 \ln 3} \rightarrow y - 3 = \frac{1}{27\ln 3}(x-27)$

$\rightarrow \boxed{y = \frac{x}{27\ln 3} - \frac{1}{\ln 3} + 3}$

(67) $y = x^{\sin x} \quad \left(\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow \ln y = \sin x \ln x$

$\rightarrow \frac{y'}{y} = \cos x(\ln x) + \sin x\left(\frac{1}{x}\right) \rightarrow \cos x \ln x + \frac{\sin x}{x} \Big|_{(\frac{\pi}{2}, \frac{\pi}{2})}$

$\rightarrow \cancel{\cos\frac{\pi}{2}\left(\ln\frac{\pi}{2}\right)} + \frac{\sin\frac{\pi}{2}}{\pi/2} = \frac{y'}{\pi/2} \rightarrow \frac{1}{\pi/2} = \frac{y'}{\pi/2} \rightarrow y' = 1$

$y-\frac{\pi}{2}=1\left(x-\frac{\pi}{2}\right) \to \boxed{y=x}$

(71) $\int 3^x\,dx \to \frac{1}{\ln 3}\cdot 3^x + C \to \boxed{\frac{3^x}{\ln 3}+C}$

(75) $\int x(5^{-x^2})\,dx \to -\frac{1}{2}\int(-2x)(5^{-x^2})\,dx$ $u=-x^2$ $du=-2x\,dx$

$-\frac{1}{2}\int 5^u\,du \to -\frac{1}{2}\left(\frac{1}{\ln 5}\cdot 5^u\right)+C \to -\frac{1}{2}\left(\frac{5^{-x^2}}{\ln 5}\right)+C$

$\to \boxed{\frac{-1}{2\ln 5}\left(5^{-x^2}\right)+C}$

(77) $\int \frac{3^{2x}}{1+3^{2x}}\,dx$ $u=1+3^{2x}$ $du=3^{2x}\ln 3(2)$

$\frac{1}{2\ln 3}\int\frac{(2\ln 3)(3^{2x})}{1+3^{2x}}\,dx \to \frac{1}{2\ln 3}\int\frac{1}{u}\,du \to \frac{1}{2\ln 3}(\ln u)+C$

$\to \boxed{\frac{1}{2\ln 3}\left(\ln|1+3^{2x}|\right)+C}$

(81) $\int_0^1 (5^x-3^x)\,dx \to \int_0^1 5^x\,dx-\int_0^1 3x\,dx$

$\to \int_0^1 5^x\,dx \to \frac{1}{\ln 5}\,5^x \to \frac{5^x}{\ln 5} \to \left[\frac{5^1}{\ln 5}-\frac{5^0}{\ln 5}\right] \to \frac{5}{\ln 5}-\frac{1}{\ln 5}$

$\to \boxed{\frac{4}{\ln 5}}$ $-\int_0^1 3^x\,dx \to -\frac{3^x}{\ln 3} \to \left[\frac{-3^1}{\ln 3}+\frac{3^0}{\ln 3}\right] \to$

$\frac{-3}{\ln 3}+\frac{1}{\ln 3} \to \boxed{\frac{-2}{\ln 3}} \to \boxed{\frac{4}{\ln 5}-\frac{2}{\ln 3}}$

5.6 HW [redacted]

③ $\sin^{-1}\left(\frac{1}{2}\right)$ finding x → $\sin(?) = \frac{1}{2} \to \boxed{\frac{\pi}{6}}$

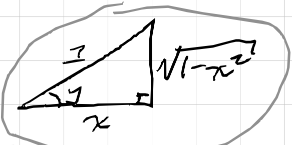
⑤ $\cos^{-1}\left(\frac{1}{2}\right) \to \cos(?) = \frac{1}{2} \to \frac{2\pi}{6} \to \boxed{\frac{\pi}{3}}$

⑦ $\tan^{-1}\frac{\sqrt{3}}{3} \to \tan(?) = \frac{\sqrt{3}}{3} \to \frac{1}{\sqrt{3}} \to \frac{1}{2}\cdot\frac{2}{\sqrt{3}} \to \boxed{\frac{\pi}{6}}$

⑪ $\cos^{-1}(-0.8) \approx 2.50$

⑫ $\sin^{-1}(-0.39) \approx -0.40$

⑮ $y = \cos^{-1}x, \quad 0 < y < \pi/2$

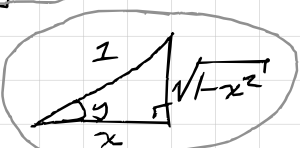


$x^2 + b^2 = 1$
$\to 1 - x^2 = b^2$
$b = \sqrt{1-x^2}$

cos y → $\cos(\cos^{-1}x) \to \boxed{\cos y = x}$

⑰ tan y

$\to \cos^{-1}x \to \tan(\cos^{-1}(x))$



$\boxed{\tan y = \frac{\sqrt{1-x^2}}{x}}$

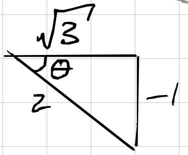
⑲ sec y

$\to \sec(\cos^{-1}(x)) \to \frac{1}{\cos}$

$\boxed{\sec y = \frac{1}{x}}$

㉓ ⓐ $\cot\left[\arcsin\left(-\frac{1}{2}\right)\right]$

$= \frac{\cos x}{\sin x} \to \frac{\sqrt{3}/2}{-1/2} \to \frac{\sqrt{3}}{2}\cdot\frac{2}{-1}$

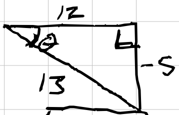


$1 + b^2 = 4 \to b^2 = 3 \to b = \sqrt{3}$

$\to \boxed{-\sqrt{3}}$

⑥ $\csc\left[\tan^{-1}\left(-\frac{5}{12}\right)\right]$

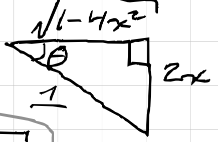
$\frac{1}{\sin} \to \frac{1}{\frac{-5}{13}} \to \boxed{\frac{13}{-5}}$



$\to 144 + 25 = h^2 \to 169 = h$
$h = \sqrt{169} = 13$

㉕ $\cos\left[\sin^{-1}(2x)\right]$

$\to \frac{\sqrt{1-4x^2}}{1} \to \boxed{\sqrt{1-4x^2}}$



$\to 2x^2 + b^2 = 1 \to b^2 = 1 - 4x^2$
$\to b = \sqrt{1-4x^2}$

㉝ $\sin^{-1}(3x-\pi)=\frac{1}{2} \rightarrow 3x-\pi=\sin\left(\frac{1}{2}\right)$

$\rightarrow 3x=\sin\left(\frac{1}{2}\right)+\pi \rightarrow \boxed{x=\frac{\left[\sin\left(\frac{1}{2}\right)+\pi\right]}{3}}$

㊴ $f(x)=2\sin^{-1}(x-1) \rightarrow 2\left[\frac{1}{\sqrt{1-(x-1)^2}}\right] \rightarrow \boxed{\frac{2}{\sqrt{2x-x^2}}}$

㊺ $g(x)=\frac{\sin^{-1}(3x)}{x}$ $high'=\frac{3}{\sqrt{1-9x^2}}$

$\rightarrow \frac{x\left(\frac{3}{\sqrt{1-9x^2}}\right)-\sin^{-1}(3x)(1)}{x^2} \rightarrow \boxed{\frac{\frac{3x}{\sqrt{1-9x^2}}-\sin^{-1}(3x)}{x^2}}$

㊾ $y=\overbrace{(2x)(\cos^{-1}x)}^{1}-\overbrace{(2\sqrt{1-x^2})}^{2} \rightarrow \left(\cancel{\frac{-2x}{\sqrt{1-x^2}}}+2\cos^{-1}(x)\right)$

$\cancel{+\frac{2x}{\sqrt{1-x^2}}}$

1: $\left(2x\left(\frac{-1}{\sqrt{1-x^2}}\right)+2(\cos^{-1}x)\right)$

2: $-2(1-x^2)^{1/2} \rightarrow -2\left(\frac{1}{2}\right)(1-x^2)^{-1/2}(-2x)$

$\rightarrow +\frac{2x}{\sqrt{1-x^2}}$ $\rightarrow \boxed{2\cos^{-1}(x)}$

(53) $y=\overbrace{x\sin^{-1}x}^{1}+\overbrace{\sqrt{1-x^2}}^{2}$

$\rightarrow$ 1: $x\left(\frac{1}{\sqrt{1-x^2}}\right)+1(\sin^{-1}(x)) \rightarrow \cancel{\frac{x}{\sqrt{1-x^2}}}+\sin^{-1}(x)-\cancel{\frac{x}{\sqrt{1-x^2}}}$

2: $\cancel{\frac{1}{2}}(1-x^2)^{-1/2}(-\cancel{2}x)$ $\rightarrow \boxed{\sin^{-1}(x)}$

(59) $y=2\sin^{-1}x$ @ $\left(\frac{1}{2},\frac{\pi}{3}\right)$

$y'=\frac{2}{\sqrt{1-x^2}} \rightarrow y'\Big|_{\frac{1}{2}}=\frac{2}{\sqrt{1-\left(\frac{1}{2}\right)^2}} \rightarrow \frac{2}{\sqrt{1-\frac{1}{4}}} \rightarrow \frac{2}{\sqrt{3/4}}\cdot\frac{2}{2}$

$\rightarrow \frac{4}{\sqrt{3/4\cdot 4}} \rightarrow \frac{4}{\sqrt{3}}=m \rightarrow y-\frac{\pi}{3}=\frac{4}{\sqrt{3}}\left(x-\frac{1}{2}\right) \rightarrow$

$y=\frac{4}{\sqrt{3}}x-\frac{2}{\sqrt{3}}+\frac{\pi}{3} \rightarrow \boxed{y=\frac{4\sqrt{3}}{3}x-\frac{2\sqrt{3}}{3}+\frac{\pi}{3}}$

$\boxed{61}$

$$y = \tan^{-1}\left(\frac{x}{2}\right) \text{ @ } \left(2, \frac{\pi}{4}\right)$$

$$\rightarrow \frac{1/2}{1+\frac{x^2}{2^2}} \rightarrow \frac{1}{2} \cdot \frac{1}{1+\frac{x^2}{2^2}} \rightarrow \frac{1}{2} \cdot \frac{1}{1+x^2/4}$$

$$\rightarrow \frac{1}{2+x^2/2} \rightarrow \frac{2}{4+x^2}\Big|_2 \rightarrow \frac{2}{4+4} \rightarrow \frac{2}{8} \rightarrow \boxed{\frac{1}{4}}$$

$$\rightarrow y - \frac{\pi}{4} = \frac{1}{4}(x-2) \rightarrow \boxed{y = \frac{1}{4}x - \frac{1}{2} + \frac{\pi}{4}}$$

$\boxed{69}$ $f(x) = (\sec^{-1}x) - x$

$$f'(x) = \frac{1}{|x|\sqrt{x^2-1}} - 1 \rightarrow 0 = \frac{1}{|x|\sqrt{x^2-1}} - 1$$

$$\rightarrow 1 = \frac{1}{|x|\sqrt{x^2-1}} \rightarrow |x|\sqrt{x^2-1} = 1 \rightarrow x^2(x^2-1) = 1$$

$$\rightarrow x^4 - x^2 = 1 \rightarrow \overset{q}{x^4} - x^2 - \overset{p}{1} = 0 \qquad t^2 - t - 1 = 0$$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} \rightarrow y = \frac{1 \pm \sqrt{1+4}}{2} \rightarrow y = \frac{1 \pm \sqrt{5}}{2}$$

$$\rightarrow x^2 = \frac{1 \pm \sqrt{5}}{2} \rightarrow x = \pm\sqrt{\frac{1+\sqrt{5}}{2}} \rightarrow \pm 1.272$$

$$f(1.272) = \sec^{-1}(1.272) - 1.272 \rightarrow -0.606$$

$$f(-1.272) = \sec^{-1}(-1.272) + 1.272 \rightarrow 3.747$$

$$\boxed{\begin{array}{l} \text{min}: (1.272, -0.606) \\ \text{max}: (-1.272, 3.747) \end{array}}$$

$\boxed{77}$ $(x^2 + x\tan^{-1}y = y - 1)\frac{dy}{dx}$ @ $\left(-\frac{\pi}{4}, 1\right)$

$$\rightarrow 2x + \left(x\left(\frac{1}{1+y^2}\right)\frac{dy}{dx} + \arctan y\right) = \frac{dy}{dx}$$

$$\rightarrow 2x + \arctan y = y' - \frac{x}{1+y^2}y' \rightarrow$$

$$2x + \arctan y = \left(1 - \frac{x}{1+y^2}\right)y' \rightarrow y' = \frac{2x + \tan^{-1}y}{1 - \frac{x}{1+y^2}}$$

$$y'\left(-\frac{\pi}{4}, 1\right) \rightarrow \frac{-\frac{\pi}{2} + \tan^{-1}(1)}{1 - \frac{-\pi/4}{1+1}} \rightarrow \frac{-\frac{\pi}{2} + \frac{\pi}{4}}{1 + \frac{\pi}{8}} \rightarrow \frac{\frac{-2\pi}{4} + \frac{\pi}{4}}{1 + \frac{\pi}{8}}$$

$$\frac{\left(-\frac{\pi}{4}\right)^{8}}{\left(+\frac{\pi}{8}\right)\vec{F}} \quad \boxed{\frac{-2\pi}{8+\pi} = m}$$

$$y-1 = \frac{-2\pi}{8+\pi}\left(x+\frac{\pi}{4}\right) \rightarrow \boxed{y = \frac{-2\pi}{8+\pi}x - \frac{\pi^2}{16+2\pi} + 1}$$

5.7 HW [redacted]

⑤ $\int \frac{1}{\sqrt{1-(x+1)^2}}\,dx$ $u = x+1$ $du = dx$ $a = 1$

$\arcsin^{-1}$?

$\rightarrow \arcsin \frac{x+1}{1} + C \rightarrow \boxed{\arcsin(x+1) + C}$

⑦ $\frac{1}{2}\int \frac{t\,(2)}{\sqrt{1-t^4}}\,dt \rightarrow$ $u = x^2$ $du = 2x\,dx$ $a = 1$

$\arcsin$?

$\rightarrow \frac{1}{2} \arcsin \frac{x^2}{1} + C \rightarrow \boxed{\frac{1}{2} \arcsin t^2 + C}$

⑫ $\int \frac{2\,(3)}{(3)x\sqrt{9x^2-25}}\,dx$ $u = 3x$ $du = 3\,dx$ $a = 5$

arcsec?

$\xrightarrow{2} \left(\frac{1}{5} \text{arcsec} \frac{|3x|}{5}\right) + C \rightarrow \boxed{\frac{2}{5} \text{arcsec} \frac{|3x|}{5} + C}$

⑬ $\int \frac{\sec^2 x}{\sqrt{25-\tan^2 x}}\,dx$ $u = \tan x$ $du = \sec^2 x\,dx$ $a = 5$

$\sin^{-1}$?

$\rightarrow \boxed{\arcsin \frac{\tan x}{5} + C}$

$u = \sqrt{x}$ $x = u^2$ $dx = 2u\,du$ $a = 1$

$du = \frac{1}{2\sqrt{x}}\,dx$

⑯ $\int \frac{3}{2\sqrt{x}(1+x)}\,dx \rightarrow$

$a = 1$ $u = u$

$\int \frac{3}{2u(1+u^2)} \cdot 2u\,du \rightarrow 3\int \frac{3}{1+u^2}\,du$

$\xrightarrow{3} \left(\frac{1}{1} \arctan\left(\frac{u}{1}\right) + C\right) \rightarrow 3\arctan(u) + C$

$\rightarrow \boxed{3\arctan(\sqrt{x}) + C}$

⑰ $\int \frac{x-3}{x^2+1}\,dx \rightarrow$ $u = x^2+1$ $du = 2x\,dx \rightarrow \frac{1}{2}\int \frac{2x}{x^2+1}\,dx$

$\frac{1}{1}$

$3\int \frac{1}{x^2+1}\,dx$

$\rightarrow \boxed{\frac{1}{2} \ln|x^2+1| - 3\arctan x + C}$

$u = x$ $a = 1$

⑲ $\int \frac{x+5}{\sqrt{9-(x-3)^2}}\,dx \rightarrow (x-3)+8$

$\rightarrow \int \frac{(x-3)}{\sqrt{9-(x-3)^2}}\,dx + 8\int \frac{1}{\sqrt{9-(x-3)^2}}\,dx$

$\rightarrow \underbrace{\int \frac{u}{\sqrt{9-u^2}}\,du} + \underbrace{8\int \frac{1}{\sqrt{9-u^2}}\,du}_{*}$

$\rightarrow b=9-u^2 \quad db=-2u\,du \rightarrow -\frac{1}{2}\int b^{-1/2}\,db$

$\rightarrow -\frac{1}{2}\left[2b^{1/2}\right]+c \rightarrow -\frac{1}{2}\left[2\sqrt{9-u^2}\right]+c$

$*\ 8\left[\arcsin\frac{u}{3}\right]+c \rightarrow \boxed{-\sqrt{9-(x-3)^2}+8\arcsin\frac{x-3}{3}+c}$

㉑ $\int_0^{1/6} \frac{3}{\sqrt{1-9x^2}}\,dx \quad u=3x \quad du=3\,dx \quad a=1$

$\sin^{-1}$?

$\rightarrow \left[\arcsin 3x\right]_0^{1/6} \rightarrow \left[\arcsin\left(\frac{1}{2}\right)-\arcsin(0)\right]$

$\rightarrow \left[\frac{\pi}{6}-0\right] \rightarrow \boxed{\frac{\pi}{6}}$

㉓ $\frac{1}{2}\int_0^{\sqrt{3}/2} \frac{1(2)}{1+4x^2}\,dx \rightarrow u=2x \quad du=2\,dx \quad a=1$

$\rightarrow \frac{1}{2}\left[\arctan 2x\right]_0^{\sqrt{3}/2} \rightarrow \frac{1}{2}\left[\arctan(\sqrt{3})-\arctan(0)\right]$

$\rightarrow \frac{1}{2}\left[\frac{\pi}{3}-0\right] \rightarrow \boxed{\frac{\pi}{6}}$

㉛ $\int_0^{1/\sqrt{2}} \frac{\arcsin x}{\sqrt{1-x^2}}\,dx \quad u=\arcsin x \quad du=\frac{1}{\sqrt{1-x^2}}\,dx$

$\rightarrow \left[\frac{\arcsin^2 x}{2}\right]_0^{1/\sqrt{2}} \rightarrow \left[\frac{(\arcsin(1/\sqrt{2}))(\arcsin(1/\sqrt{2}))}{2} - \frac{(\arcsin(0))(\arcsin(0))}{2}\right] \rightarrow \left[\frac{\arcsin^2\left(\frac{\sqrt{2}}{2}\right)}{2}\right] \rightarrow \left(\frac{\left(\frac{\pi}{4}\right)^2}{2}\right]$

$\rightarrow \frac{\pi^2}{16}\cdot\frac{1}{2} \rightarrow \boxed{\frac{\pi^2}{32}}$

(33) $\int_0^2 \frac{1}{x^2-2x+2}\,dx$ $\left(x^2-2+\underline{1}\right)+2-\underline{1}$ $\left(\frac{b}{2}\right)^2$

$\to \int_0^2 \frac{1}{(x-1)^2+1}\,dx$ $(x-1)^2+1$ $\to \left(\frac{-2}{2}\right)^2 \to \boxed{1}$

$\to \left[\arctan(x-1)\right]_0^2 \to \left[\arctan(1) - \arctan(-1)\right]$

$\to -\left(-\frac{\pi}{4}\right)$

$\to \left[\frac{\pi}{4}+\frac{\pi}{4}\right] \to \boxed{\frac{\pi}{2}}$

(45) $\int_1^3 \frac{1}{\sqrt{x}(1+x)}\,dx$ $u=\sqrt{x}$ $u^2=x$ $2u\,du=dx$

$1+x=1+u^2$ $du=\frac{1}{2\sqrt{x}}\,dx$

$\int_{\sqrt{1}}^{\sqrt{3}} \frac{2u\,du}{u(1+u^2)} = \int_1^{\sqrt{3}} \frac{2}{1+u^2}\,du$ $\frac{\sqrt{3}}{2}\cdot\frac{2}{1}$

$\to 2\left[\tan u\right]_1^{\sqrt{3}} \to 2\left[\tan\sqrt{3} - \tan 1\right]$

$\to 2\left[\frac{\pi}{3}-\frac{\pi}{4}\right] \to 2\left[\frac{4\pi}{12}-\frac{3\pi}{12}\right] \to 2\left[\frac{\pi}{12}\right] \to \boxed{\frac{\pi}{6}}$

(49) ⓐ $\int\sqrt{x-1}\,dx \to u=x-1 \to \boxed{\frac{2x^{3/2}}{3}+C}$

ⓑ $\int x\sqrt{x-1}\,dx$ $u=\sqrt{x-1}$ $x=u^2+1$ $dx=2u\,du$

$\to \int(u^2+1)(u)(2u)\,du \to \int(u^3+u)(2u)\,du \to 2\int(u^4+u^2)\,du$

$\to 2\left[\frac{u^5}{5}+\frac{u^3}{3}\right]+C \to 2\left[\frac{3u^5}{15}+\frac{5u^3}{15}\right]+C$

$\to \frac{2}{15}u^3\left[3u^2+5\right]+C \to \left(\frac{2}{15}(x-1)^{3/2}\right)\left(3(x-1)+5\right)+C$

$\to \boxed{\frac{2(x-1)^{3/2}}{15}(3x+2)+C}$

$\frac{1}{2\sqrt{x-1}}\,dx$

ⓒ $\int\frac{x}{\sqrt{x-1}}\,dx$ $u=\sqrt{x-1}$ $x=u^2+1$ $du=2u\,du$

$\int\frac{x}{\sqrt{x-1}}\,dx = \int\frac{u^2+1}{u}(2u)\,du \to 2\int u^2+1\,du \to 2\left[\frac{u^3}{3}+u\right]+C$

$\to \frac{2}{3}u(u^2+3)+C \to \boxed{\frac{2}{3}\sqrt{x-1}\,(x+2)+C}$

(71) $\int \frac{1}{3x\sqrt{9x^2-16}}\,dx = \frac{1}{4}\,\text{arcsec}\,\frac{3x}{4} + c$

$\rightarrow u = 3x \quad du = 3dx \rightarrow a = 4$ False,

$\rightarrow \frac{1}{3}\int \frac{3}{3x\sqrt{9x^2-16}}\,dx \rightarrow \boxed{\frac{1}{12}\,\text{arcsec}\,\frac{|3x|}{4} + C}$

8.7 HW Q: [redacted], [redacted], [redacted], [redacted], [redacted] [redacted]

③ $$\boxed{\lim_{x\to\infty} x^5 e^{-x/100} = 0}$$

x	1	10	10^2	10^3	10^4	10^5
$f(x)$	0.99	90483.74	3.67E9	4.54E10	3.72E-24	0

⑦ $$\lim_{x\to 6} \frac{\sqrt{x+10}-4}{x-6} \to \frac{\sqrt{6+10}-4}{x-6} \to \frac{\sqrt{16}-4}{6-6} \to \frac{4-4}{0} \to \frac{0}{0}$$

ⓐ

$$\to \text{unit IE 3:} \quad \frac{\sqrt{x+10}-4}{x-6}\,\frac{(\sqrt{x+10}+4)}{(\sqrt{x+10}+4)} \to \frac{x+10-16}{(x-6)(\sqrt{x+10}+4)}$$

$$\to \frac{\cancel{x-6}}{(\cancel{x-6})(\sqrt{x+10}+4)} \to \frac{1}{\sqrt{x+10}+4} \to \lim_{x\to 6} \frac{1}{\sqrt{6+10}+4} \to \boxed{\frac{1}{8}}$$

ⓑ $$\lim_{x\to 6} \frac{\frac{1}{2\sqrt{x+10}}}{1} \to \frac{1}{2\sqrt{x+10}} \to \frac{1}{2\sqrt{16}} \to \frac{1}{2(4)} \to \boxed{\frac{1}{8}}$$

⑮ $$\lim_{x\to 0^+} \frac{e^x-(1+x)}{x^3} \to \frac{e^0-(1+0)}{0^3} \to \frac{0}{0}$$

$$\xrightarrow{L'H} \frac{e^x-1}{3x^2} \quad \frac{e^0-1}{3(0)^2} \to \frac{1-1}{0} \to \frac{0}{0}$$

$$\xrightarrow{L'H} \frac{e^x}{6x} \to \frac{1}{0} \to \boxed{\lim_{x\to 0^+} f(x) = \infty}$$

⑲ $$\lim_{x\to 0} \frac{\sin 3x}{\sin 5x} \to \frac{\sin(0)}{\sin(0)} \to \frac{0}{0}$$

$$\xrightarrow{L'H} \frac{(\cos 3x)3}{(\cos 5x)5} \to \frac{(\cos(0))3}{(\cos(0))5} \to \frac{(1)3}{(1)5} \to \boxed{\frac{3}{5}}$$

㉗ $$\lim_{x\to\infty} \frac{x^3}{e^{x/2}} \to \frac{\infty^3}{e^{\infty/2}} \to \frac{\infty}{\infty}$$

$$\xrightarrow{L'H} \frac{3x^2}{e^{x/2}(\frac{1}{2})} \to \frac{3(\infty)^2}{e^{\infty/2}(\frac{1}{2})} \to \frac{\infty}{\infty(\frac{1}{2})} \to \frac{\infty}{\infty} \qquad \frac{e^{x/2}}{2}$$

$$\xrightarrow{L'H} \frac{6x}{\frac{1}{4}e^{x/2}} \to \frac{6(\infty)}{\frac{1}{4}e^{(\infty/2)}} \to \frac{\infty}{\infty} \xrightarrow{L'H} \frac{6}{\frac{1}{8}e^{x/2}} \to \frac{1}{\infty} \to \boxed{0}$$

㊴ $\lim_{x \to 0} \frac{\tan^{-1}(x)}{\sin(x)} \to$ $\tan(?) \to 0 \to 0/1$

$\tan^{-1}(0) \to ?$

$$\frac{u'}{1+u^2}$$

$\to \frac{0}{0}$

$$\to^{L'H} \lim_{x \to 0} \frac{\frac{1}{1+x^2}}{\cos x} \to \frac{\left(\frac{1}{1+0^2}\right)}{1} \to \frac{\frac{1}{1}}{1} \to \boxed{1}$$

6.1 HW [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted],

[redacted]

③ $y' = \frac{2xy}{x^2-y^2}$ $x^2+y^2 = Cy \rightarrow 2x + 2y\frac{dy}{dx} = C\frac{dy}{dx}$

$\rightarrow 2yy' - Cy' = -2x \rightarrow y'(2y-C) = -2x$

$\rightarrow y' = \frac{-2x}{2y-C} \rightarrow y' = \frac{-2xy}{2y^2-Cy} \rightarrow y' = \frac{-2xy}{\cancel{2}y^2-(x^2+\cancel{y^2})}$

$\rightarrow y' = \left(\frac{-2xy}{y^2-x^2}\right)(-1) \rightarrow y' = \frac{2xy}{x^2-y^2}$

✓

⑨ $y = \sin x\cos x - \cos^2 x$ $2y + y' = 2\sin(2x) - 1;\ y(\frac{\pi}{4}) = 0$

$(-\cos x)(\cos x)$

$y' = (\sin x(-\sin x) + (\cos x)(\cos x)) + (\sin x)(\cos x) + (-\cos x)(-\sin x)$

$-\sin x^2 = \cos^2 x - 1$

$y' = -\sin^2 x + \cos^2 x + 2\sin x\cos x$

$\sin 2x = 2\sin x\cos x$

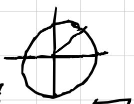
$\rightarrow 2(\sin x\cos x - \cos^2 x) + (-\sin^2 x + \cancel{\cos^2 x} + 2\sin x\cos x) = 2\sin(2x) - 1$

$\rightarrow 2\sin x\cos x - \cos^2 x - \sin^2 x + 2\sin x\cos x = 2\sin(2x) - 1$

$\rightarrow \sin 2x - \cos^2 x - \sin^2 x + \sin 2x = 2\sin 2x - 1$

$2\sin 2x - 1 = 2\sin 2x - 1$

Init. Cond.: $y(\frac{\pi}{4}) = 0$



$\rightarrow \sin\frac{\pi}{4}\cos\frac{\pi}{4} - \cos^2(\frac{\pi}{4}) \rightarrow \cancel{\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}} - \cancel{\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}} = \boxed{0}$

⑪ $y = 4e^{-6x^2}$ $y' = -12xy$ $y(0) = 4$

$\rightarrow y' = 4e^{-6x^2}(-12x) \rightarrow -48xe^{-6x^2} \rightarrow y' = -12x(4e^{-6x^2})$

$\frac{d}{dx}[e^u] \rightarrow e^u u'$

$u' = -12x$ $\rightarrow y' = \boxed{-48xe^{-6x^2}}$

Init. Cond.: $(0,4)$

$\rightarrow 4e^{-6(0)^2} = 4 \rightarrow 4e^0 \rightarrow \boxed{4=4}$

⑬ $y=3\cos x$ $\quad y^{(4)}-16y=0$ $\quad \frac{d}{dx}[\cos x]=-\sin x$

$\frac{d}{dx}[\sin x]=\cos x$

$y'=-3\sin x$

$y''=-3\cos x$

$y'''=3\sin x$

$y^{(4)}=3\cos x$

$\rightarrow 3\cos x-16(3\cos x)=0$

$3\cos x-48\cos x=0$

$-45\cos x\neq 0$

Function is not a solution

⑯ $y=3\sin(2x)$ $\quad y^{(4)}-16y=0$

$y'=6\cos 2x \rightarrow y''=-12\sin 2x \quad y'''=-24\cos 2x$

$y^{(4)}=48\sin 2x \quad 48\sin 2x-16(3\sin 2x)=0$

$\rightarrow \boxed{48\sin 2x-48\sin 2x=0}$

㉑ $xy'-2y=x^3e^x$ $\quad \frac{d}{dx}[x^a]=ax^{a-1}$

$y=x^2$

$y'=2x$

$\rightarrow x(2x)-2x^2=x^3e^x \rightarrow 2x^2-2x^2=x^3e^x$

$\rightarrow 0\neq x^3e^x$ Not a solution

㊶ $\frac{dy}{dx}=6x^2 \rightarrow dy=6x^2\,dx \quad \int dy=\int 6x^2\,dx$

$\rightarrow \boxed{y=2x^3+c}$

㊹ $\frac{dy}{dx}=\frac{e^x}{4+e^x} \rightarrow dy=\frac{e^x}{4+e^x}\,dx$

$\int dy=\int\frac{e^x}{4+e^x}\,dx \quad u=4+e^x \quad du=e^x\cdot dx$

$\boxed{y=\ln|4+e^x|+c}$

㊽ $\frac{dy}{dx}=\tan^2x \rightarrow \int dy=\int\tan^2x\,dx \rightarrow \int\sec^2x-1\,dx$

$\rightarrow \boxed{y=\tan x-x+c}$

(53)

x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
y'	-4	undefined	0	1	4/3	2

$\frac{2(-4)}{2} = -4$ $\frac{2(-2)}{0} =$ undefined

$\frac{2(0)}{4} = 0$

$\frac{2(2)}{4} \rightarrow 1$ $\frac{2(4)}{6} \rightarrow \frac{4}{3}$ $\frac{8(2)}{8} \rightarrow 2$

(57) $-2\cos(2x) + C$ (58) $\frac{1}{2}\sin x + C$ (59) $\frac{-2}{e^{2x}} + C$ (60) $\ln|x| + C$

↳ (b) ↳ (c) ↳ (d) ↳ (a)

6.2 HW [redacted], [redacted], [redacted]; [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted]
[redacted]

① $\frac{dy}{dx} = x+3 \rightarrow dy = x+3\,dx \rightarrow \int dy = \int x+3\,dx$

$\rightarrow \boxed{y = \frac{x^2}{2} + 3x + C}$

③ $\frac{dy}{dx} = y+3 \rightarrow \int \frac{dy}{y+3} = \int dx \rightarrow \ln|y+3| = x + C_1$

$\rightarrow y+3 = e^{x+C_1} \rightarrow y+3 = e^x \cdot e^{C_1} \rightarrow y+3 = Ce^x$

$\rightarrow \boxed{y = Ce^x - 3}$

⑤ $y' = \frac{5x}{y} \rightarrow \frac{dy}{dx} = \frac{5x}{y} \rightarrow \int y\,dy = \int 5x\,dx$

$\rightarrow \frac{y^2}{2} = \frac{5x^2}{2} + C \rightarrow y^2 = 5x^2 + C$

$\rightarrow \boxed{y^2 - 5x^2 = C}$

⑩ $xy + y' = 100x \rightarrow \frac{dy}{dx} - xy = 100x \quad P(x) = x \quad Q(x) = 100x$

$\rightarrow \frac{dy}{dx} - P(x)y = Q(x) \quad \mu(x) = e^{\int x\,dx} = e^{x^2/2}$

$e^{x^2/2}\frac{dy}{dx} + xe^{x^2/2}y = 100xe^{x^2/2} \rightarrow \frac{d}{dx}\left(ye^{x^2/2}\right) = 100xe^{x^2/2}$

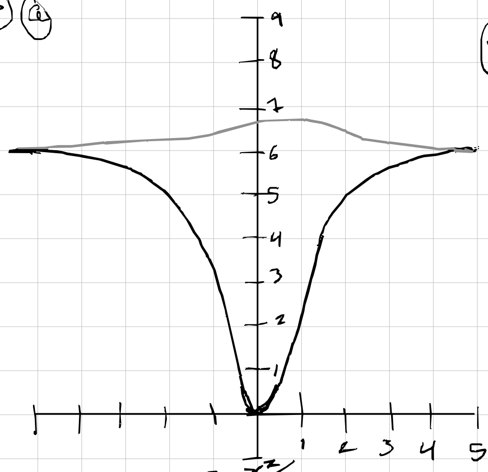
$\rightarrow \int d\left(ye^{x^2/2}\right) = \int 100xe^{x^2/2}\,dx \rightarrow ye^{x^2/2} = 100\int e^{x^2/2}x\,dx$

$u = x^2/2 \quad du = x\,dx$

$\int e^u\,du = e^u = e^{x^2/2} \rightarrow ye^{x^2/2} = 100e^{x^2/2} + C$

$\boxed{y = 100 + Ce^{-x^2/2}}$

⑬ ⓐ



ⓑ $\frac{dy}{dx} = x(6-y)$, $(0,0)$

$\rightarrow \frac{dy}{dx} = -x(y-6)$

$\rightarrow \int \frac{dy}{y-6} = \int -x\,dx$

$\rightarrow \ln|y-6| = -\frac{x^2}{2} + c$

$\rightarrow y-6 = e^{-\frac{x^2}{2}+C}$

$\rightarrow y-6 = Ce^{-x^2/2} \rightarrow y = 6+Ce^{-x^2/2}\Big|_{(0,0)} \rightarrow 0 = 6+Ce^{-0^2/2}$

$\rightarrow 0 = 6+C \rightarrow \boxed{C=-6} \rightarrow \boxed{y = 6-6e^{-x^2/2}}$

⑲ $\frac{dN}{dx} = kN$; $t=0$, $N=250$; $t=1$, $N=400$; $t=4$, $N=?$

$\rightarrow \int \frac{1}{N}\,dN = \int k\,dx \rightarrow \ln|N| = kx + c$ $\quad \boxed{t=x}$

$e^{\ln|N|} = e^{kx+c} \rightarrow N = Ce^{kx}$

$\rightarrow 250 = Ce^{k(0)} \rightarrow 250 = C \qquad 400 = Ce^{k(1)}$

$\rightarrow 400 = Ce^{k} \qquad N = Ce^{k(4)}$

$400 = 250e^{k}$

$\rightarrow 1.6 = e^{k} \rightarrow \ln 1.6 = k \approx 0.47$

$\rightarrow N = 250e^{\ln 1.6(4)} \rightarrow \boxed{N = 1638.4 \text{ @ } t=4}$

㉑ Points $(0, \frac{1}{2})$; $(5, 5)$ find $y = Ce^{kt}$

$\frac{1}{2} = Ce^{k(0)} \rightarrow \frac{1}{2} = C \qquad 5 = Ce^{k(5)} \rightarrow 5 = \frac{1}{2}e^{k(5)}$

$\rightarrow 10 = e^{k(5)} \rightarrow \ln 10 = 5k \rightarrow k = \frac{\ln 10}{5}$

$\longrightarrow \boxed{y = \frac{1}{2}e^{\frac{\ln 10}{5}t}}$

(31) ^{226}Ra HL: 1599 Initial quantity: $\boxed{7.632}$

Amount after 1,000 years: $\boxed{4.947g}$

Amount after 10,000 years: 0.1g

$$y = Ce^{kt}$$

$$\frac{1}{2} = e^{k(1599)} \to \ln 1/2 = 1599k \to k = \frac{\ln\frac{1}{2}}{1599}$$

$$\to 0.1 = Ce^{\frac{\ln\frac{1}{2}}{1599}(10,000)} \to C = \frac{0.1}{e^{\frac{\ln\frac{1}{2}}{1599}(10,000)}} = 7.632$$

$$y = 7.632e^{\frac{\ln\frac{1}{2}}{1599}(1,000)} \to 4.947$$

(33) ^{14}C HL: 5715 Init. Quant. 5g

amount after 1000 years: $\boxed{4.429g}$

amount after 10,000 years: $\boxed{1.487g}$

$$\frac{1}{2} = 1e^{k(5715)} \to \ln\frac{1}{2} = 5715k \to k = \frac{\ln\frac{1}{2}}{5715}$$

$$\to y = Ce^{kt} \to y = 5e^{\frac{\ln\frac{1}{2}}{5715}(10,000)} = 1.487$$

$$y = 5e^{\frac{\ln\frac{1}{2}}{5715}(1,000)} = 4.429$$

(35) ^{239}Pu HL: 24,100 Initial quantity: $\boxed{2.161g}$

amount after 1000 years: 2.1g

amount after 10,000 years: $\boxed{1.575g}$

$$\frac{1}{2} = 1e^{k(24,100)} \to \frac{\ln\frac{1}{2}}{24,100} = k \to 2.1 = Ce^{\frac{\ln\frac{1}{2}}{24,100}(1000)}$$

$$\to \frac{2.1}{e^{\frac{\ln\frac{1}{2}}{24100}1000}} = C = \boxed{2.161}$$

$$\to y = 2.161\, e^{\frac{\ln\frac{1}{2}}{24100}10,000} \to \boxed{1.575}$$

(37) $HL = 1599 \rightarrow \frac{1}{2} = 1e^{k1599} \rightarrow \frac{\ln\frac{1}{2}}{1599} = k$

$y = 100e^{\frac{\ln\frac{1}{2}}{1599}100} \rightarrow$ $\boxed{95.758\text{ \% remains}}$

(39) $P = 4{,}000 \quad r = 0.06$

$\boxed{\begin{array}{l}\text{Time to double: } 11.552 \text{ years} \\ \text{Amount after 10 years: } \$7288.48\end{array}}$

$A = Pe^{rt}$

$8{,}000 = 4{,}000e^{0.06t}$

$\rightarrow 2 = e^{0.06t} \rightarrow \ln 2 = 0.06t \rightarrow \frac{\ln 2}{0.06} = t = 11.552$

$A = 4{,}000e^{0.06(10)} \rightarrow \$7288.475$

(41) Initial: 750 $\boxed{\text{Annual rate: } 8.94\%}$

double time: 7.75 years $\boxed{\text{Amount after 10 years: } \$1834.37}$

$1500 = 750e^{r(7.75)} \rightarrow \frac{1500}{750} = e^{r(7.75)}$

$\rightarrow \ln\frac{1500}{750} = 7.75r \rightarrow \frac{\ln\frac{1500}{750}}{7.75} = r = 0.0894$

$\rightarrow A = 750e^{0.0894(10)} = 1834.37$

(45) $A = P(1+\frac{r}{n})^{nt} \quad r = 0.075 \quad t = 20 \quad n = 12$

$1{,}000{,}000 = P(1+\frac{0.075}{12})^{12(20)}$

$P = \frac{1{,}000{,}000}{(1+\frac{0.075}{12})^{12(20)}}$

Amount to invest $\boxed{: 224{,}174.18}$

(56) 125 @ $t=2$; 350 @ $t=4$

ⓐ initial population: $\boxed{45}$

$125 = Pe^{r(2)} \qquad 350 = Pe^{r(4)} \rightarrow \frac{350}{125} = \frac{Pe^{r4}}{Pe^{r2}}$

$\rightarrow \frac{350}{125} = \frac{e^{4r}}{e^{2r}} \rightarrow \frac{350}{125} = e^{4r-2r} \rightarrow \frac{350}{125} = e^{2r} \rightarrow \ln\frac{350}{125} = 2r$

$r = (\ln\frac{350}{125})/2 = 0.5148 \rightarrow 125 = Pe^{0.5148(2)} \rightarrow P = \frac{125}{\ln\frac{350}{125}} = \underline{44.643}$

ⓑ $\boxed{y=44.463e^{0.5148(t)}}$

ⓒ $y=44.463e^{0.5148(8)}=2732.73=\boxed{2733}$

(59) ⓐ If the population increases by a set amount every month, it can be modeled by $y=P+C(t)$
or $y=bx+C$
which is a linear function. (constant slope)

ⓑ If the population is increasing at a set rate, it can be modeled by the compounding model:

$$y=\text{Principle population}\left(1+\frac{\text{rate of growth}}{12\text{ months}}\right)^{12\text{ months (years)}}$$
$$=P\left(1+\frac{r}{n}\right)^{nt}$$

which is an exponential function.

6.3 HW [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted]

③ $x^2+5y\frac{dy}{dx}=0 \to \int 5y\,dy = -\int x^2\,dx$

$\to \frac{5y^2}{2} = -\frac{x^3}{3}+C \to \frac{5y^2}{2}+\frac{x^3}{3}=C$

$\to \boxed{15y^2+2x^3=C}$

⑦ $(2+x)\frac{dy}{dx}=3y \to \int\frac{1}{y}\,dy=\int\frac{3}{2+x}\,dx$

$\to \ln|y| = 3\int\frac{1}{2+x}\,dx+C \to \ln|y| = 3\ln|2+x|+C$

$\boxed{y=C(2+x)^3}$

⑪ $\sqrt{1-4x^2}\,\frac{dy}{dx}=x \to \int 1\,dy=\int\frac{x}{\sqrt{1-4x^2}}\,dx$

$y=\int x(1-4x^2)^{-1/2}\,dx \quad +C$

$u=1-4x^2$

$u'=-8x \quad du=-8x\,dx$

$\to y=-\frac{1}{8}\int u^{-1/2}\,du \quad +C$

$\to y=\frac{-1}{8}2u^{1/2} \quad +C \to \boxed{y=\frac{-\sqrt{1-4x^2}}{4}+C}$

⑫ $\sqrt{x^2-16}\,\frac{dy}{dx}=11x \to 1\,dy=\frac{11x}{\sqrt{x^2-16}}\,dx$

$\to y=11x(x^2-16)^{-1/2}\,dx \quad +C$

$u=x^2-16$

$u'=2x$

$du=2x\,dx$

$\to y=\frac{11}{2}\int u^{-1/2}\,dx+C$

$\to y=\frac{11}{2}(2u^{1/2})+C \to y=\frac{11}{2}(2(x^2-16))+C$

$\to \boxed{y=11(x^2-16)+C}$

⑬ $y\ln x - xy' = 0 \to y\ln x - x\frac{dy}{dx} = 0$

$y\ln x = x\frac{dy}{dx} \Rightarrow \int \frac{\ln x}{x}\,dx = \int \frac{1}{y}\,dy$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\to \ln|y| = \int u\,du + C$

$\to \ln|y| = \frac{\ln x^2}{2} + c \to \boxed{y = Ce^{\frac{\ln x^2}{2}}}$

⑮ $yy' - 2e^x = 0$ @ $(0,3)$

$\to y\frac{dy}{dx} - 2e^x = 0 \to \int y\,dy = \int 2e^x\,dx$

$\to \frac{y^2}{2} = 2e^x + C \to y^2 = 4e^x + c$

$\to 3^2 = 4e^0 + c \to 9 = 4 + C \to 5 = C \to \boxed{y^2 = 4e^x + 5}$

⑱ $2xy' - \ln x^2 = 0 \to 2x\frac{dy}{dx} - \ln x^2 = 0$ @ $(1,2)$

$\to \int 1\,dy = \int \frac{\ln x^2}{2x}\,dx \to y = \int \frac{\ln x}{x}\,dx + C$

$u = \ln x$

$u' = \frac{1}{x}$

$du = \frac{1}{x}\,dx$

$\to y = \int u\,du + C \to y = \frac{\ln x^2}{2} + C$

$\to 2 = \frac{(\ln 1)^2}{2} + C \to 2 = C$

$\to \boxed{y = \frac{\ln x^2}{2} + 2}$

㉓ $dP - kP\,dt = 0$ @ $P(0) = P_0 \to (0,0)$

$\to dP = kP\,dt \to \int \frac{1}{P}\,dP = \int k\,dt$

$\to \ln|P| = kt + C \to P = Ce^{kt}$

$P(0) \to Ce^{k(0)} \to C = 0 \to \boxed{P = P_0e^{kt}}$

㉝ ⓐ $\frac{dy}{dx} = k(y-4)$

ⓑ graph "a", because $\frac{dy}{dx} \neq 0$ on $y=0$

㉞ ⓐ $\frac{dy}{dx} = k(x-4)$

ⓑ graph "b", because $\frac{dy}{dx} = 0$ on $x=4$

㊲ $\frac{dy}{dt} = ky$

$\frac{1}{2} = 1e^{k(1599)} \rightarrow \frac{\ln(\frac{1}{2})}{1599} = k$

$y = 100e^{\frac{\ln(\frac{1}{2})}{1599}(50)} \rightarrow \boxed{y = 97.856\%}$

7.1 HW Non-Calc: [redacted], [redacted], [redacted], [redacted], [redacted], [redacted]; Calc: [redacted], [redacted]

① $y_1 = x^2 - 6x$ $y_2 = 0$

$$\boxed{\int_0^6 (0-(x^2-6x))\,dx}$$

$0 = x^2 - 6x$

$\rightarrow 0 = x(x-6)$

$x = 0 \quad x = 6$

② $y_1 = x^2 + 2x + 1$ $y_2 = 2x+5$

$$\boxed{\int_{-2}^{2} ((2x+5)-(x^2+2x+1))\,dx}$$

$x^2 + \cancel{2x} + 1 = \cancel{2x} + 5$

$x^2 - 4 = 0 \rightarrow x^2 = 4$

$x = \pm 2$

③ $y_1 = x^2 - 4x + 3$ $y_2 = -x^2 + 2x + 3$

$$\boxed{\int_0^3 ((-x^2+2x+3)-(x^2-4x+3))\,dx}$$

$x^2 - 4x + \cancel{3} = -x^2 + 2x + \cancel{3}$

$2x^2 - 6x = 0$

$2x(x-3) = 0$

$x = 0 \quad x = 3$

④ $y_1 = x^2$ $y_2 = x^3$

$$\boxed{\int_0^1 ((x^2)-(x^3))\,dx}$$

$x^2 = x^3$

$x^3 - x^2 = 0$

$x = 0 \quad x = 1$

⑥ $y_1 = (x-1)^3$ $y_2 = x-1$

$$\boxed{A = \int_0^1 ((x-1)^3 - (x-1))\,dx + \int_1^2 ((x-1)-(x-1)^3)\,dx}$$

$(x-1)^3 = x-1$

$\frac{(x-1)(x-1)(x-1)}{x-1} = \frac{x-1}{x-1}$

$(x-1)(x-1) = 1$

$x^2 - 2x + 1 = 1$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$

$(x-1)^3 - x + 1 = 0$

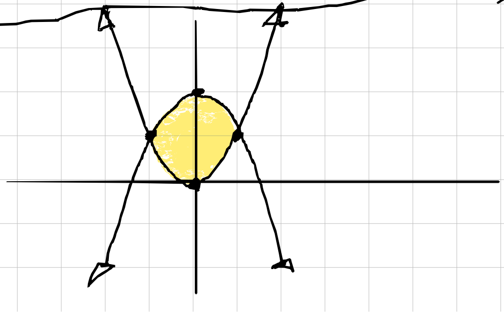
$(1-1)^3 - 1 + 1 = 0$

$0 = 0$

$x = 1$

$[0, 1, 2]$

⑧



⑲ $f(x) = x^2+2x \quad g(x) = x+2$

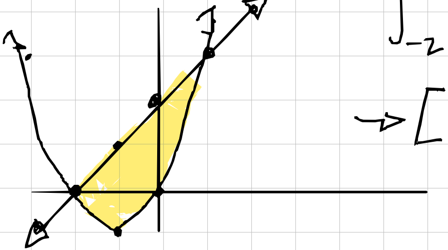
$x+2 = x^2+2x$

$0 = x^2+x-2$

$0 = (x+2)(x-1)$

$x = -2, 1$

$$A = \int_{-2}^{1} ((x+2)-(x^2+2x))\,dx$$



$$\int_{-2}^{1} (x+2-x^2-2x)\,dx$$

$$\rightarrow \left[\frac{x^2}{2}+2x-\frac{x^3}{3}-x^2\right]_{-2}^{1}$$

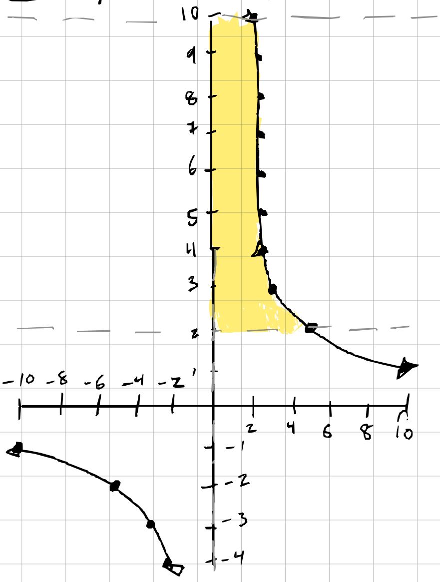
$$\rightarrow \left(\frac{1}{2}+2-\frac{1}{3}-1\right)-\left(\frac{-2^2}{2}+(-2(2))-\frac{-2^3}{3}-(-2)^2\right)$$

$$\rightarrow \frac{1}{2}+2-\frac{1}{3}-1-\left(2-4+\frac{8}{3}-4\right)$$

$$\frac{1}{2}\cancel{+2}-\frac{1}{3}-1\cancel{-2}+4-\frac{8}{3}+4 \rightarrow 8-1+\frac{1}{2}-\frac{1}{3}-\frac{8}{3}$$

$$\rightarrow \frac{42}{6}+\frac{3}{6}-\frac{2}{6}-\frac{16}{6} \quad \frac{26}{6}+\frac{1}{6} \rightarrow \frac{27}{6} \rightarrow \boxed{\frac{9}{2}} \text{ or } 4.5$$

(29) $f(x) = \frac{10}{x}, \quad x=0, \quad y=2, \quad y=10$



$$A = \int_{2}^{10} \frac{10}{x}\,dx$$

$$A = 10\int_{2}^{10} \frac{1}{x}\,dx$$

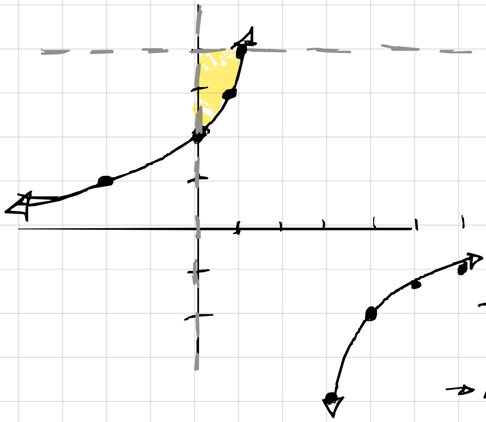
$$A = 10[\ln|x|]_{2}^{10}$$

$$A = 10[(\ln|10|)-(\ln|2|)]$$

$$A = 10 \ln\frac{10}{2} \rightarrow \boxed{10\ln 5}$$

$$\approx 16.094$$

(30) $g(x)=\frac{4}{2-x}$, $y=4$, $x=0$ $\rightarrow y=\frac{4}{2-x} \rightarrow y(2-x)=4$



$$2-x=\frac{4}{y} \rightarrow 2-\frac{4}{y}=x$$

$$A=\int_2^4\left(4-\left(2-\frac{4}{y}\right)\right)dy$$

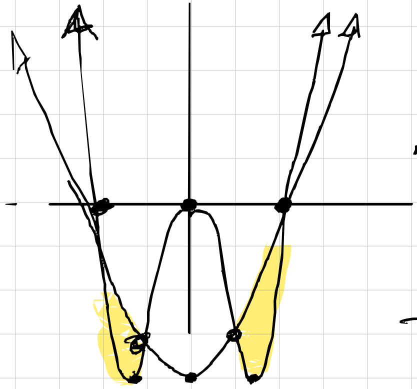
$$\rightarrow \int_2^4\left(4-2+\frac{4}{y}\right)dy$$

$$\rightarrow \left[4y-2y+4\ln|y|\right]_2^4$$

$$\rightarrow \left[(16-8+4\ln 4)-(8-4+4\ln 2)\right]$$

$$\rightarrow 8+4\ln 4-4-4\ln 2 \rightarrow \boxed{4-4\ln 2} \text{ or } 1.227$$

(33) $f(x)=x^4-4x^2$, $g(x)=x^2-4$ $\rightarrow$



$-2, -1, 1, 2$

$$A=\int_{-2}^{-1}\left((x^2-4)-(x^4-4x^2)\right)dx$$

$$+\int_1^2\left((x^2-4)-(x^4-4x^2)\right)dx$$

$$\rightarrow \boxed{\frac{44}{15}} \text{ or } 2.933$$

(35) $f(x)=\frac{6x}{x^2+1}$ $y=0$, $0\le x\le 3$

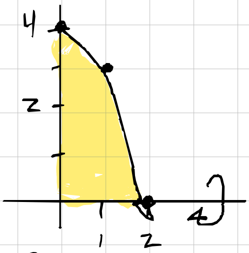
$$A=\int_0^3\left(\left(\frac{6x}{x^2+1}\right)-0\right)dx$$

$$\rightarrow \boxed{6.907}$$

HW 7.2

Q: [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted], [redacted]

② $y = 4-x^2 \quad y=0 \quad 16-x^4$

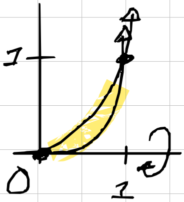


$$V = \pi\int_0^2 (4-x^2)^2\,dx \approx \boxed{53.617}$$

$$\pi\left[16x - \frac{x^5}{5}\right]_0^2 \rightarrow \pi[(32-\tfrac{32}{5})-(0)]$$

$$\rightarrow \pi\left[\frac{160}{5} - \frac{32}{5}\right] \rightarrow \pi\left(\frac{125}{32}\right) \rightarrow \frac{125\pi}{32}$$

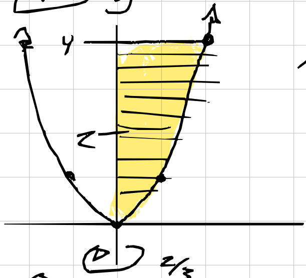
⑤ $y=x^2, \; y=x^5$



$$V = \pi\int_0^1 \left((x^2)^2 - (x^5)^2\right) dx$$

$$\approx \boxed{0.343} \text{ or } \frac{6\pi}{55}$$

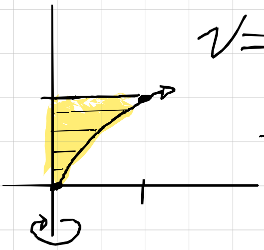
⑦ $y=x^2 \rightarrow x = \pm\sqrt{y}$



$$V = \pi\int_0^4 y\,dy \rightarrow \pi\left[\frac{y^2}{2}\right]_0^4 \rightarrow \pi\left[\frac{16}{2} - 0\right]$$

$$\rightarrow \boxed{8\pi} \text{ or } 25.133$$

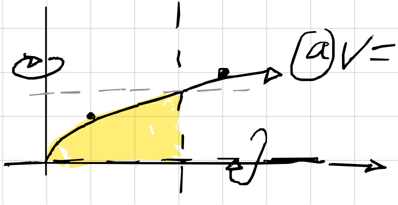
⑨ $y = x^{2/3} = \sqrt[3]{x^2} = y \rightarrow x^2 = y^3 \rightarrow x = \pm\sqrt{y^3} \rightarrow x = \pm y^{3/2}$



$$V = \pi\int_0^1 y^3\,dy \rightarrow \pi\left[\frac{y^4}{4}\right]_0^1$$

$$\rightarrow \pi\left[\frac{1}{4} - 0\right] \quad \boxed{\frac{\pi}{4}} \approx 0.785$$

⑪ $y = \sqrt{x}, \; y=0, \; x=3$



ⓐ $V = \pi\int_0^3 (x)\,dx \rightarrow \pi\left[\frac{x^2}{2}\right]_0^3$

$$\pi\left[\frac{9}{2} - \frac{0}{2}\right] \rightarrow \boxed{\frac{9\pi}{2}} \text{ or } 14.137$$

$$\boxed{b}\quad V=\pi\int_0^{\sqrt{3}}\left((3)^2-(y^2)^2\right)dy$$

$y=\sqrt{x}$

$\to y^2=x$

$y^2=3$

$y=\sqrt{3}$

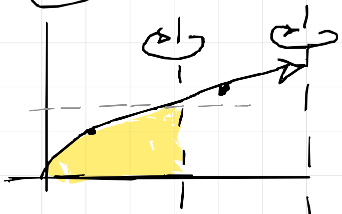
$$\to \pi\left[9x-\frac{y^5}{5}\right]_0^{\sqrt{3}}$$

$$\pi\left[\left(9\sqrt{3}-\frac{3^{5/2}}{5}\right)-0\right]$$

$$\to 9\sqrt{3}\pi-\frac{3^{5/2}\pi}{5}\approx\boxed{39.178}$$

$\boxed{c}$

$3-x^2$



$$\pi\int_0^{\sqrt{3}}(3-y^2)^2\,dy \to \pi\left[9x-2y^3+\frac{y^5}{5}\right]_0^{\sqrt{3}}$$

$$(9-6y^2+y^4)$$

$$\longrightarrow \pi\left[9\sqrt{3}-2(3)^{3/2}+\frac{3^{5/2}}{5}\right]$$

$$\longrightarrow 9\sqrt{3}\pi-2(3)^{3/2}\pi+\frac{3^{5/2}\pi}{5}\approx\boxed{26.119}$$

$\boxed{d}$

$$V=\pi\int_0^{\sqrt{3}}\left((6-y^2)^2-(3)^2\right)dy \to \pi\int_0^{\sqrt{3}}\left(\underset{27}{\cancel{36}}-12y^2+y^4\cancel{-9}\right)dy$$

$$\to \pi\left[27y-4y^3+\frac{y^5}{5}\right]_0^{\sqrt{3}} \to \pi\left[\left(27\sqrt{3}-4\sqrt{27}+\frac{\sqrt{243}}{5}\right)-(0)\right]$$

$$\to 27\sqrt{3}\pi-4\sqrt{27}\pi+\frac{\sqrt{243}\pi}{5}\approx\boxed{91.415}$$

$\boxed{17}$ $y=\frac{3}{1+x}$, $y=0$, $x=0$, $x=3$ $\circlearrowleft y=4$

$$V=\pi\int_0^3\left(4^2-\left(4-\frac{3}{1+x}\right)^2\right)dx$$

$$\int\frac{9}{(1+x)^2}dx$$

$$9\int(1+x)^{-2}dx$$

$$9\left[\frac{(1+x)^{-1}}{}\right]$$

$$\to -\frac{9}{1+x}$$

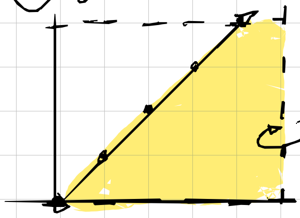
$$\to \pi\int_0^3\left(16-\left(16-\frac{24}{1+x}+\frac{9}{(1+x)^2}\right)\right)dx$$

$$\to \pi\int_0^3\left(\frac{24}{1+x}-\frac{9}{(1+x)^2}\right)dx \to \pi\left[24\ln|1+x|+\frac{9}{1+x}\right]_0^3$$

$$\to \pi\left[\left(24\ln|4|+\frac{9}{4}\right)-(9)\right] \to 24\pi\ln 4+\frac{9\pi}{4}-9\pi$$

$$\approx\boxed{83.318}$$

⑲ $y=x,\ y=0,\ y=4,\ x=5,\ \circlearrowleft x=5 \quad \to x=y$



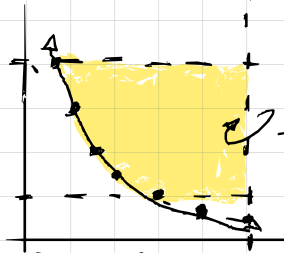
$$V=\pi\int_0^4 (5-y)^2\,dy \to \pi\int_0^4 (25-10y+y^2)\,dy$$

$$\to \pi\left[25y-5y^2+\frac{y^3}{3}\right]_0^4$$

$$\to \pi\left[\left(100-80+\frac{64}{3}\right)-0\right] \to \pi\left[20+\frac{64}{3}\right] \to \pi\left[\frac{124}{3}\right]$$

$$\to \frac{124\pi}{3} \approx \boxed{129.852}$$

㉒ $xy=3,\ y=1,\ y=4,\ x=5;\ \circlearrowleft x=5 \qquad y=\frac{3}{x} \quad x=\frac{3}{y}$



$$V=\pi\int_1^4\left(\left(\frac{3}{y}-5\right)^2\right)dy \to \pi\int_1^4\left(\frac{9}{y^2}-10\frac{3}{y}+25\right)dy$$

$$\to \pi\left[-\frac{9}{y}-30\ln|y|+25y\right]_1^4$$

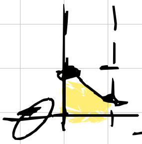
$$\pi\left[\left(-\frac{9}{4}-30\ln 4+100\right)-(-9+25)\right]$$

$9\int y^{-2} \to -\frac{9}{y}$

$$\to \pi\left[-\frac{9}{4}-30\ln 4+84\right] \to \frac{-9\pi}{4}-30\pi\ln 4+84\pi$$

$$\approx \boxed{126.170}$$

㉗ $y=e^{-x},\ y=0,\ x=0,\ y=1,\ \circlearrowleft x\text{-axis}$

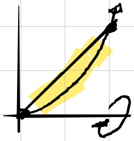


$$V=\pi\int_0^1 (e^{-x})^2\,dx \to \pi\int_0^1 (e^{-2x})\,dx$$

$$\to \left[-\frac{\pi}{2}e^{-2x}\right]_0^1 \to \frac{-\pi}{2}e^{-2}-\left(-\frac{\pi}{2}\right) \to \frac{-\pi}{2}e^{-2}+\frac{\pi}{2}$$

$$\to \frac{\pi}{2}(1-e^{-2}) \to \approx \boxed{1.358}$$

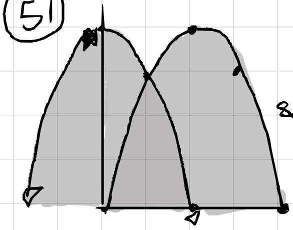
㊸ $y=x^2,\ y=x$



$$V=\pi\int_0^1\left(x^2-(x^2)^2\right)dx \to \pi\int_0^1 (x^2-x^4)\,dx$$

$$\to \pi\left[\frac{x^3}{3}-\frac{x^5}{5}\right]_0^1 \to \pi\left[\left(\frac{1}{3}-\frac{1}{5}\right)-0\right] \to \frac{2\pi}{15} \approx \boxed{0.419}$$

(51)

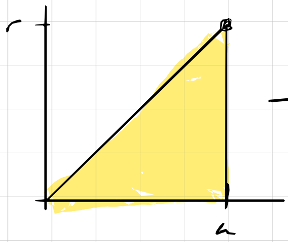


Both $y=4x-x^2$ & $y=4-x^2$ have the same area before revolving & integrating, so the volume of both are the same.

(54) a) iii b) iv c) i d) ii

(59) Verify that V(right, circular cone) = $\frac{1}{3}\pi r^2 h$

Use disk method →



$$V=\pi\int_0^h\left(\frac{r^2}{h^2}x^2\right)dx$$

$$\rightarrow\left[\frac{r^2\pi}{3h^2}x^3\right]_0^h \rightarrow \frac{r^2\pi}{3h^2}h^3=\frac{1}{3}\pi r^2 h$$